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Surplus Identification with Non-Linear Returns

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Abstract: We present evidence from two experiments designed to quantify the impact of cognitive constraints on consumers' ability to identify surpluses. Participants made repeated forced-choice decisions about whether products conferred surpluses, comparing one or two plainly perceptible attributes against displayed prices. Returns to attributes varied in linearity, scale and relative weight. Despite the apparent simplicity of this task, in which participants were incentivised and able to attend fully to all relevant information, surplus identification was surprisingly imprecise and subject to systematic bias. Performance was unaffected by monotonic non-linearities in returns, but non-monotonic non-linearities reduced the likelihood of detecting a surplus. Regardless of the shape of returns, learning was minimal and largely confined to initial exposures. Although product value was objectively determined, participants exhibited biases previously observed in subjective discrete choice, suggesting common cognitive mechanisms. These findings have implications for consumer choice models and for ongoing attempts to account for cognitive constraints in applied microeconomic contexts.

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1 Introduction

Benefit accrues to economic agents through the process of exchange when what is acquired is worth more than what is given up. A prerequisite for making gains from trade, therefore, is that agents can accurately identify surpluses. This paper presents evidence that cognitive limitations disrupt surplus identification, even when agents attend fully to a small volume of complete information. The findings have implications for consumer choice models that aim to incorporate cognitive constraints and the response of firms to such constraints.

To spot surpluses agents must integrate information about product attributes and prices. Returns to different attributes and combinations of attributes may vary in scale and linearity. Furthermore, in a dynamic economy, prices, attributes and preferences evolve. Learning new associations between them is an ongoing process for agents seeking to maximise utility; a process that is subject to at least occasional error. Most of us have goods tucked away at home that did not live up to our assessment when we bought them, as well as cherished possessions that we value far more than originally anticipated. Doubtless such outcomes are, in part, due to stochastic variation in unobservable product quality, but they may also be caused by failure to integrate observable information accurately when deciding to purchase, i.e., to imprecise identification of surpluses. Thus, the human ability to integrate non-linear and, often, novel attribute information is fundamental to economic exchange and market efficiency. It is this ability that we examine experimentally.

Empirical assessment of how accurately people identify surpluses is, of course, hampered by the subjective and unobservable nature of preferences. Previous investigations have mainly focused on markets where relative surpluses can be (nearly) objectively defined, for example where goods are (effectively) homogeneous (Grubb, 2009; Wilson and Waddams Price, 2010; Choi et al., 2010). The assumption is that where consumers opt to pay more for essentially the same product they are failing to identify surpluses accurately. The evidence, reviewed briefly below, suggests that consumers in some markets fail to identify the highest surpluses, with mixed evidence regarding learning. Such findings are generating an increasingly rich theoretical literature, with contrasting approaches to the formalisation of cognitive constraints within models of consumer decision-making (Woodford, 2014), discrete choice (Matejka and McKay, 2015) and, increasingly, industrial organisation (Grubb, 2015).

The present paper takes an alternative empirical approach. We employ experimental designs adapted from the study of perceptual detection, keeping product attributes and associated returns under complete experimental control. As described in detail below, we incentivise participants to identify objectively defined surpluses based on a price and just one or two directly observable product attributes. The method measures the magnitude of surplus required for reliable detection when returns to attributes differ in linearity, scale and relative weight. The results reveal how the accuracy of consumers' surplus detection varies as a function of the shape of attribute returns. This variation in accuracy is illuminating with regard to the concept of a "complex product", which is extensively used but rarely defined.

Our experimental results suggest that descriptively accurate choice models might focus on imprecision in the integration of information when consumers must map incommensurate internal scales to compare attributes and prices. We show that this imprecision is large and varies systematically with the shape of attribute returns. When people map attribute magnitudes to prices they are able to detect a surplus reliably only when it corresponds to a substantial proportion of the price or attribute range. The experiments reveal that surplus detection is largely unaffected by whether returns are linear or non-linear, provided they are monotonic, but that it deteriorates when returns are non-monotonic. Moreover, despite considerable exposure to the product and repeated feedback, decisions are subject to persistent biases, with only a modest role for learning. We conclude that inaccuracy of surplus identification on this scale is likely to be an important determinant of consumer outcomes in different product markets, with associated implications for microeconomic models. The magnitude of the errors we uncover is consistent with Luce's (1959) view that decision-making is essentially a process of probabilistic choice.

The paper is organised as follows. Section 2 briefly reviews evidence on the ability of consumers to identify surpluses, together with relevant theoretical developments. Section 3 describes the "Surplus Identification" (S-ID) Task. Section 4 describes Experiment 1, which investigated surplus identification for single-attribute products with varying degrees of diminishing returns. Section 5 presents Experiment 2, which increased the complexity of the attribute-price relationship via a second attribute and non-monotonic associations. Section 6 explores additional hypotheses regarding learning and biases, which are important for the generalisability of our results. Section 7 concludes.

2 Literature review: Limits to Surplus Identification

2.1 Empirical Evidence

A growing empirical literature documents how consumers, in at least some markets, miss out on surpluses. Apparent "mistakes" can be identified without the need to make strong assumptions about preferences where offerings are (effectively) homogeneous and so the maximisation of consumer surplus reduces to a process of cost minimisation, assuming positive marginal utility of money. For instance, substantial numbers of mobile phone consumers fail to choose the lowest cost tariff for their personal pattern of usage from a limited number of non-linear price plans (Grubb, 2009; Lambrecht and Skiera, 2006), probably because of overconfidence and inattention (Grubb and Osborne, 2015). Similar evidence has been obtained for two- and three-part tariffs in domestic electricity markets by Wilson and Waddams Price (2010), who conclude that "many of the choices are consistent with genuine decision error or inattention" (p.665). Consumers in these studies faced non-linear price structures. Some evidence suggests that non-linear returns to attributes may be difficult for consumers to assess, including non-linear measures of fuel efficiency in the car market (Larrick and Soll, 2008) and returns to compound interest in financial services markets (Lusardi and Mitchell, 2011).

Other evidence suggests that consumers sometimes fail to assess surpluses accurately because they overweight largely irrelevant but prominently advertised product attributes, or underweight price components. This can be as simple as overweighting premium brands and, hence, failing to benefit from cheaper generic medicines (Bronnenberg et al., 2015). In relation to indexed mutual funds, laboratory experiments (Choi et al., 2010) and data on fund flows (Barber et al., 2005) suggest overweighting of past performance information and underweighting of fees. This may have implications for market structure since, despite the homogeneity of the good, price dispersion for indexed funds rivals that for actively-managed ones (Hortacsu and Syverson, 2003). Grubb (2015) reviews additional evidence for consumer errors when prices are multidimensional.

2.2 Constrained Capacity in Consumer Choice Models

Since at least Simon (1955), some microeconomic models have sought to incorporate *bounded rationality* due to "physiological and psychological limitations" (p. 101). One approach is to append additional capacity constraints to an otherwise standard utility maximisation model (Lipman, 1995).[1] More recent models of *rational inattention* exploit information theory to recast economic agents as input-processing vehicles subject to finite Shannon capacity limits (Sims, 1998, 2003; Sims et al., 2010).[2] Agents allocate limited attention optimally across information sources.[3]

Contrastingly, Woodford (2014) applies a capacity constraint to the processing of information over time rather than across multiple sources, modelling the agent as an optimal sensor facing a discrete choice, where the benefit of waiting to observe more signals trades off against the probabilistic cost of an option becoming unavailable. Again, a fundamental limit is imposed on the mutual information between the input state and an output signal, but the central trade-off to be optimised differs. Because Woodford (2014) incorporates time, his model makes predictions about decision response times that we test via the S-ID task.

Models of rational inattention provide generalisable explanations for stochastic errors, but little guidance on the extent of inaccuracy or variation in errors across contexts. Other approaches maintain the optimisation framework but introduce a non-optimal "behavioural" assumption to explain why, in specific circumstances, surpluses may not be accurately identified. Kőszegi and Szeidl (2013) propose that utility is "focus weighted", with greater weight placed on attributes that differ most between available options. Bordalo et al. (2013) make the similar, but distinct, assumption that additional weight is given to options with attributes that stand out relative to other options. Implicit is a more general assumption about cognitive capacity, namely that there is either too much immediate attribute information or too great a reliance on memory for attribute weights. The result is context-specific calibration and, hence, non-optimal attribute weighting and inaccurate surplus identification.

The above models represent advances in our understanding of how agents might miss out on surpluses. Despite differences of approach, they possess important commonalities. Cognitive capacity is constrained with respect to the volume of relevant information that can be processed, or attended to, simultaneously. However, the models make only limited predictions about the magnitude and variability of errors. Yet the source of inaccuracies, their scale and variation across

[1] Alternative approaches dispense entirely with the concept of optimisation in favour of heuristics (see Gigerenzer and Selten (2002)). See also Harstad and Selten (2013) and Rabin (2013).

[2] While Shannon Entropy has been primary used to model information-processing costs in the rational inattention framework, more generalized cost functions exist (see Caplin and Dean (2013)).

[3] When each option is equally likely to be chosen *ex ante*, the probability of choosing an item reduces to the standard multinomial logit formula (Matejka and McKay, 2015).

markets have further theoretical significance. Most obviously, small individual errors may imply trivial deviations from optimal outcomes, but large errors could imply significant welfare losses. Consumer errors have implications for industrial organisation too. An increasingly developed theoretical literature explores the implications when the potential for errors is endogenous both to the extent of consumer search and to firms' decisions to exploit opportunities to obfuscate quality or, more commonly, price (Gabaix and Laibson, 2006; Carlin, 2009; Wilson and Waddams Price, 2010; Ellison and Wolitzky, 2012; Grubb, 2015). While space does not permit a thorough treatment here, two points are notable. First, there is empirical evidence of firm obfuscation (Ellison and Ellison, 2009; Muir et al., 2013). Second, the precise source of consumer error can be important to firms' decisions to obfuscate, to equilibrium price dispersion and to the relationship between prices and marginal costs (Gabaix and Laibson, 2006; Gaudeul and Sugden, 2012; Gabaix et al., 2015; Spiegler, 2014).

2.3 Learning to Identify Surpluses

The view that preferences develop over time through mistakes and feedback forms the basis of Plott's (1996) *discovered preferences* hypothesis. Applied to surplus identification, it suggests that consumer mistakes reduce with feedback and market experience as consumers learn what they like.

Evidence of consumers' capacity for learning is mixed. Costly mistakes can lead consumers to make better choices, for instance in local telephony and credit card markets (Miravete, 2003; Agarwal et al., 2005, 2008). However, in telecommunications and electricity markets consumer mistakes have been shown to persist (Lambrecht and Skiera, 2006) and to repeat over multiple switches (Wilson and Waddams Price, 2010). This mixed evidence may imply that different psychological mechanisms are key to different instances where consumers misjudge surpluses. The nature of feedback may matter. A credit card customer who does not initially consider the possibility of incurring a particular penalty, may factor this possibility in after a single application of the fee. In contrast, if surpluses are misjudged in a market with multiple product attributes or price components, agents may remain unaware of the missed surplus and, consequently, error prone.

Even given sufficient information and feedback, evidence from psychology suggests that the potential for learning may ultimately be limited. Dating back to Miller (1956) and beyond, an accumulation of experimental evidence shows that individuals have difficulty with cognitive tasks that require them to process, simultaneously, more than around four separate "chunks" of information (Cowan, 2000). A substantial literature also investigates the ability to learn functional relationships, typically by requiring individuals to predict outcomes based on the magnitudes of multiple cues. Broadly speaking, simple monotonic associations are learned best, but performance deteriorates as cues are added and non-linear functions become more complex (Busemeyer et al., 1997).

3 The Surplus Identification (S-ID) Task

3.1 Conceptual framework

The empirical research described above highlights markets where consumers misjudge surpluses. Models of consumer choice and industrial organisation have been amended to account for the implied limits to cognitive capacity. Yet a richer understanding of the prevalence, magnitude, persistence and cause of consumers' failure to identify surpluses is required. The experiments described here offer a fresh approach, which centres on obtaining full experimental control over the scale, linearity and relative weight of product attributes. This section introduces the logic of the experimental task.

The Surplus Identification (S-ID) Task is an experimental paradigm devised by Lunn and Bohacek (2015). It measures consumers' ability to integrate attribute information from first principles, beginning with simple products that possess a single clearly observable characteristic, then increasing complexity in an experimentally controlled fashion. In this way, it permits the empirical isolation of those aspects of the attribute-price relationship that have a negative impact consumers' surplus identification, allowing controlled investigation of what truly constitutes a "complex" product. This contrasts with previous approaches that test for consumer biases in specific markets. Instead the task isolates cognitive limitations that are likely to generalise across markets.

The S-ID task blends techniques from studies of perception, psychophysics and experimental economics. Participants are presented with novel, computer-generated products, consisting of one or more attributes and a price tag. We refer to these products as "hyperproducts", because they permit complete experimental control over the attribute-price hyperspace. Our hyperproducts possess several key design features. They are objects with intuitive value, but which are nevertheless new to participants, thereby minimising any influence of prior beliefs or preferences. The attributes consist of standard perceptual stimuli, the relative magnitudes of which previous studies of perception have shown can be discriminated with high accuracy. This is important, because we are interested in how accurately consumers integrate information to gauge surplus, not how accurately they discriminate perceptual magnitudes.

The S-ID task tests surplus identification via a two-alternative forced-choice (2AFC) task, as routinely employed in studies of perceptual detection (Macmillan and Creelman, 2004). The attribute-price relationship is set by the experimenters and participants are incentivised to learn it from examples and practice with feedback. Prices, attribute magnitudes and, hence, surpluses, then vary over a succession of trials. On each trial, the consumer judges the surplus to be either positive or negative, responding via one of two buttons, then receiving feedback. The S-ID task therefore simulates the process of encountering a new product, making repeated purchase decisions and receiving feedback about the surplus gained (or lost). Thus, the task circumvents the problem of unobservable preferences by incentivising participants to identify surpluses that are objectively defined and expressed in monetary terms. With this level of experimental control, surplus can be varied from trial to trial and a precise statistical estimate of the probability of detection obtained. Moreover, because the data are collected as a time-series, the extent and speed of learning can be observed.

The S-ID task borrows from experimental economics by offering a clear incentive to adopt the preference function set by the experimenters. In the experiments described below, a tournament incentive was used. The most accurate performers received a significant monetary reward. Hence, each participant's unambiguous incentive was to learn and to apply the objective function that related attribute magnitudes to prices as quickly and accurately as possible.

Several properties of the S-ID task are important to note. First, it is not a valuation or pricing task: participants do not generate estimates of value or price, but decide only whether a surplus is present. This feature is by design. When consumers decide on purchases they do not typically generate numeric assessments. Second, participants are not placed under time pressure but complete the sequence of trials in their own time, generally taking longer on more difficult trials. While it is an empirical question as to whether surplus identification might improve with longer deliberation, as with real purchases the participant's incentive is to take longer to decide if they feel they need to. Third, the unambiguous incentive to adopt the predetermined attribute-price relationship imposes preferences upon participants. We contend that this process mimics learning to apply subjective preferences formed through experience of real products. As described below, the data offer empirical support, since we observe biases in our data that parallel those observed in subjective discrete choice experiments, implying common cognitive mechanisms.

Overall, we find that participants readily grasp the idea of the S-ID task and compete for rewards. The level of experimental control over repeated decisions generates rich data and permits analyses that are beyond what is typically possible in most consumer choice experiments.

3.2 Main Measures

The SI-D Task is able to distinguish between inaccuracy arising from bias and from imprecision. To quantify these effects, we employ two concepts from psychophysics: the "point of subjective equality" (PSE) and the "just noticeable difference" (JND). Both correspond to the parameters of a logistic "psychometric function" fitted to the bivariate data, where the binary dependent variable is whether the participant responded that the surplus was positive and the continuous exogenous variable is the magnitude of the surplus. When the surplus is very high, participants always respond that it is positive; when it is very low they always respond that it is negative. Intervening levels produce probabilistic responses.

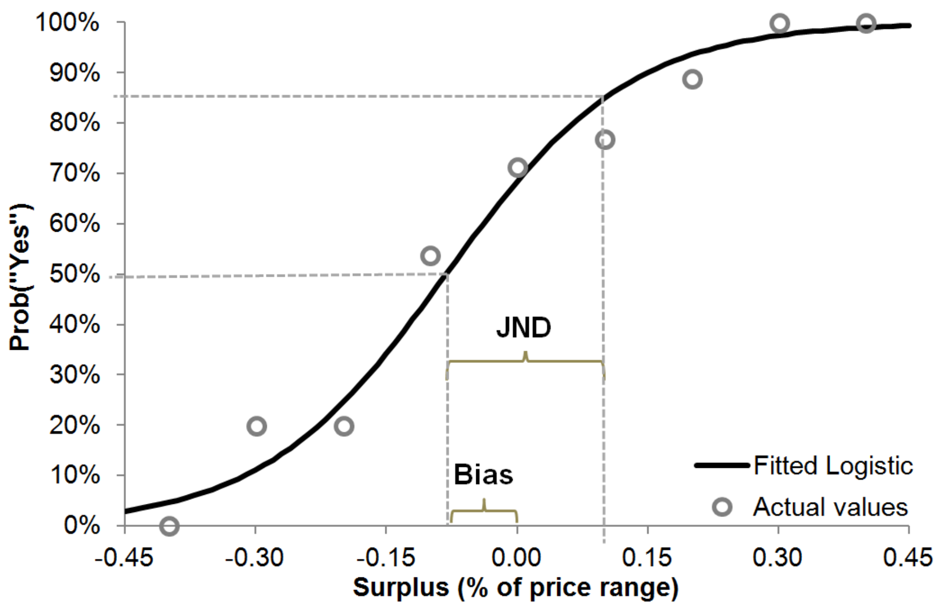
The PSE and JND correspond to the location and slope respectively of the best fitting psychometric function. Figure 1 illustrates the PSE and JND for data from a participant in one condition in our first experiment. The PSE estimates the surplus at which the participant responded with a probability of 0.5. Thus, the negative PSE indicates overestimation of surplus; a positive PSE would indicate underestimation. One JND is then the difference in surplus required for the probability that the participant detected a positive (or negative, since the psychometric function is symmetric) surplus to rise from 0.5 to 0.86, which equates to one standard deviation of the underlying logistic distribution. For an unbiased subject, it estimates how much surplus is needed to identify the surplus correctly 86% of the time.

We model the probability of responding "Yes" to a surplus by the standard logistic formula, [4]

$$\Pr(\text{"Yes"}) = \frac{\exp(\theta_0 + \theta_1 Surplus(\%))}{1 + \exp(\theta_0 + \theta_1 Surplus(\%))}. \tag{1}$$

The estimated coefficients for this sample data are $\widehat{\theta}_0 = 0.77$ and $\widehat{\theta}_1 = 9.50$, giving a PSE of $\widehat{\theta}_0/\widehat{\theta}_1 = -8.1\%$ and a JND of $\pi/(\widehat{\theta}_1\sqrt{3}) = 19.1\%$. Hence this participant perceived zero surplus when in fact there was a negative surplus of 8.1%. The JND indicates than the participant required an increase in surplus equivalent to 19.1% of the price range to detect it with 86% reliability.

Figure 1: Sample data from experiment 1



The primary analyses measure changes in the PSE and JND by condition and over time. Estimating individual psychometric functions by participant and condition – as in equation (1) – entails loss of statistical efficiency, so we instead estimate mixed effects logistic (MEL) models to the complete data. Individuals are assumed to vary randomly in bias and precision, allowing for a correlation between the two. Tests for changes by condition in the underlying fixed effects parameters constitute the main hypothesis tests of interest. Relative to an individual-level analysis, this approach improves statistical power (Moscatelli et al., 2012).

[4]The use of a logistic is standard in psychophysics and micro-founded by models of rational inattention in situations where responses are equally likely ex ante (see Matejka and McKay (2015)), as in the S-ID task.

4 Experiment I

The most simple test of surplus identification with non-linear returns involves a single-attribute product with a monotonic, continuous relationship between attribute and price. Experiment 1 aimed to generate baseline measures of accuracy and learning when facing diminishing returns to attributes of varying degrees.

4.1 Method

Consumers from the Dublin area (N=36) were recruited through a market research company, balanced by gender (19 female), age (M=36.1; SD=12.8) and working status (61% employed). Each received a €20 participation fee. Participants were informed that the most accurate performer in every ten would win a €50 shopping voucher. (In addition to the main sample of 36, another 26 participants completed a pilot experiment to test how accurately the product attributes could be discriminated, as briefly described at the beginning of Section 4.2).

The three "hyperproducts" used were Golden Eggs, Victorian Lanterns and Mayan Pyramids - products with intuitive value that participants would be highly unlikely to have valued or traded previously. Each hyperproduct could vary on two attributes (see Figure 2).[5] For the Golden Egg, overall size and the fineness of the surface texture (highest spatial frequency component) varied. For the Victorian Lantern, the ratio of an inner blue flame to the overall flame and the number of sparks emitted from the base varied. For the Mayan Pyramid, the width of the staircase and the mouldiness of the bricks varied. On any one experimental run, however, participants attended to only a single attribute, which uniquely determined the surplus. Each of the six attributes therefore matched a standard visual discrimination task that has been thoroughly investigated previously: discrimination of size, texture, ratio, numerosity, interval and colour saturation respectively.

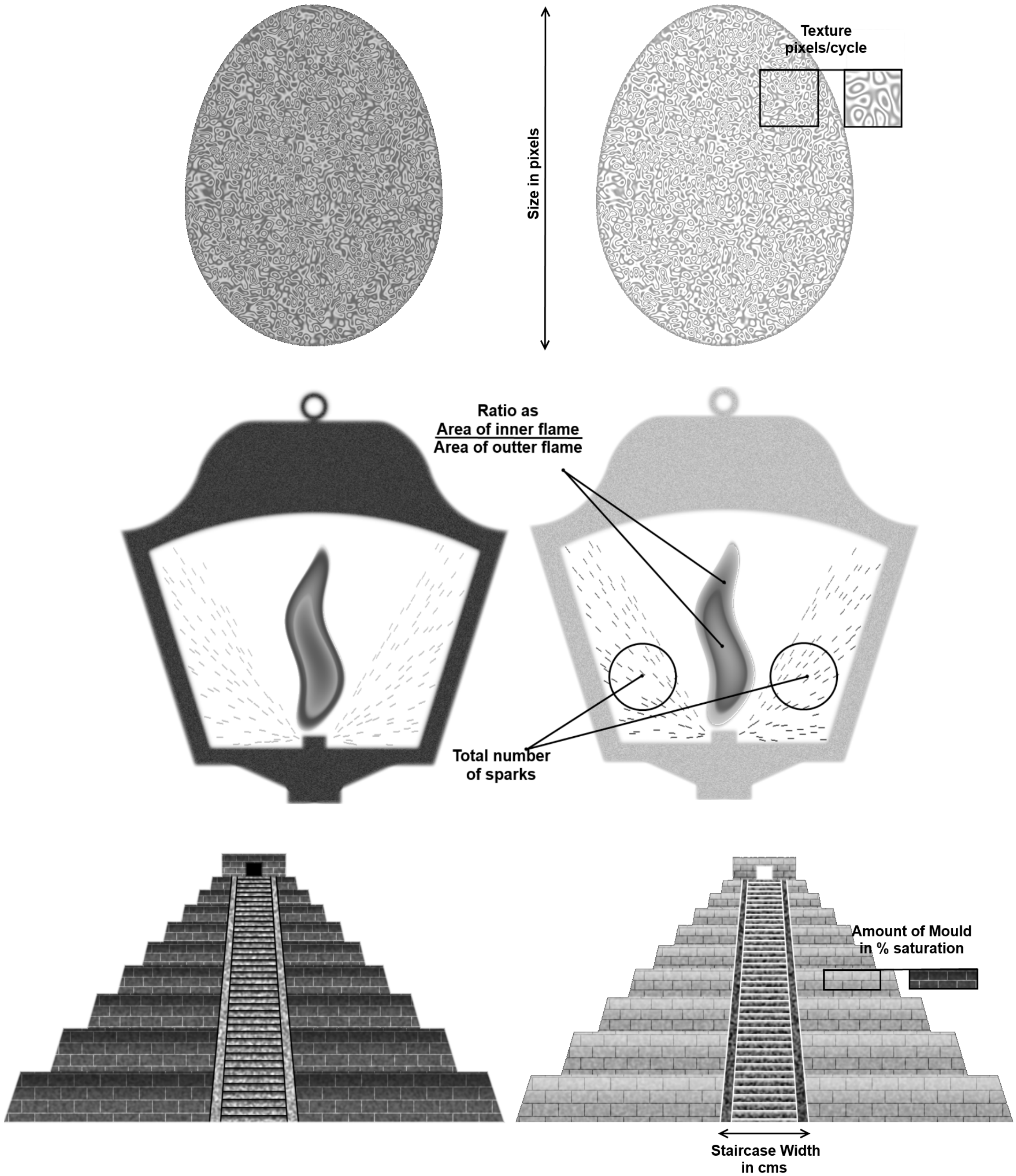
The objectively defined price of the product on each trial depended on one attribute, as follows:

$$P_{hct} = \beta_{h0} + \beta_{h1} x_t^{\alpha(c)}, \qquad x_t \in [0,1], \qquad \alpha(c) \in \left\{1, \frac{2}{3}, \frac{1}{3}\right\} \tag{2}$$

where P_{hct} is the price of hyperprudct h on trial t for condition c in Euro, β_{h0} is the minimum price a hyperproduct can take, β_{h1} scales attribute magnitudes onto the price range for a given

[5] For a detailed description of the generation process for the three hyperproducts used, please consult supplementary material.

Figure 2: Hyperproducts used in Experiment 1



hyperproduct, x_t is the normalised magnitude of the relevant attribute on trial t, and $\alpha(c)$ defines the degree of diminishing returns. The value of $\alpha(c)$ was therefore the main manipulation of interest and took one of three values - 1 (linear, $c = 1$), 2/3 (moderate diminishing returns, $c = 2$), and 1/3 (rapid diminishing returns, $c = 3$). For each hyperproduct, the price range was always the same: €180-420 for the Golden Egg; €7-35 for the Lantern; and €23,000-172,000 for the pyramid.

Participants completed the following experimental procedure.[6] Before each of six (two attributes x three products) pseudo-randomised experimental runs, they were shown systematic examples of attribute magnitudes and corresponding prices, followed by eight "practice" trials. They then undertook 72 "test" trials. On each trial, $t = 1, ...72$, the task was always the same, to determine whether the product was worth more (surplus) or less (no surplus) than the displayed price. Participants proceeded at their own speed, with brief breaks between experimental runs and a longer refreshment break after the third run. The session, including break, typically lasted just under an hour.

The main experimental conditions, $c \in \{1, 2, 3\}$, corresponded to the three values of $\alpha(c)$, pseudo-randomised across participants and attributes such that each participant completed two runs for each condition, with the proviso that the two attributes of each hyperproduct corresponded to different values of $\alpha(c)$. In addition, to check whether perceptual constraints played any role, the range, $r \in$ {high,low}, of attribute magnitudes was manipulated. In the "high" condition, the maximum available perceptible range (based on pilot studies) was used, while in the "low" condition this range was halved. Notwithstanding the non-linearity, one unit of the attribute was therefore worth twice as much in the low range condition. Hence there was a total of three conditions, c, three hyper-products, h, and two range conditions r, leading to 18 possible combinations that were pseudo-randomised across our 36 subjects.

The surplus, Δ_t, on each trial was selected using an adaptive procedure. Each run of 72 test trials consisted of nine blocks of eight. Within a block, Δ_t corresponded to four positive and four equal and opposite negative surpluses with a constant separation, {7δ, 5δ, 3δ, δ, -δ, -3δ, -5δ, -7δ}, where δ was a proportion of the mean price, presented in a random order. If the participant responded correctly on seven or eight trials, δ was reduced for the next block; if six were correct, δ

[6]Brevity is emphasised here. A detailed description to permit complete replication is available as supplementary materials, along with all information, consent, demographic and debriefing forms, experimenter script and instructions.

remained unchanged; if less than six, δ was increased. Thus, the difficulty of the task adapted to the participant's performance to aid efficient estimation of their capability.[7]

For each trial, t, the hyperproduct and displayed price were selected as follows. A "display price", P_{ht}^d, was drawn at random from a uniform distribution such that $P_{ht}^d \in [\beta_{h0} + |\Delta_t|, \beta_{h0} + \beta_{h1} - |\Delta_t|]$. The surplus was added to generate the "product price", i.e., $P_{ht} = P_{ht}^d + \Delta_t$. The unique solution for the relevant attribute magnitude, x_t, was then derived and set according to (2). The non-relevant attribute magnitude was selected randomly. Note that there was no correlation between the display price and the correct answer, i.e, the probability of a positive or negative surplus was always 0.5.

Both the hyperproduct and its display price remained on screen until the participant responded via one of two buttons on a response box. The participant then received three types of feedback: a green tick or a red cross indicated whether the response was correct, an auditory beep accompanied an incorrect answer, and the true P_{ht} was presented. Feedback was left on screen until the participant pressed a "NEXT" button.

4.2 Pilot Experiment

Prior to the main experiment, a pilot (N=26) checked the perceptual ability to discriminate the attribute magnitudes. The findings matter, because noise inherent in perceptual representations is not of interest here; imprecision in relating those representations to surpluses is. The pilot design was identical to that described above, except that two hyperproducts were simply presented alongside each other. One had an attribute magnitude calculated as for P_{ht}^d, the other as for P_{ht}. The participant had to determine which was superior.

Since normalised attribute magnitudes vary from zero to one, the JND can be measured as an absolute difference. Figure 3 presents mean JNDs for the six attributes, in high and low range conditions.[8] The attributes were discriminated with high accuracy, with differences of less than 0.1 perceived reliably. There was variability across the attributes, with between 10 and 31 separate levels of magnitude discernible.[9] Thus, if perceptual noise were to limit performance in Experiment

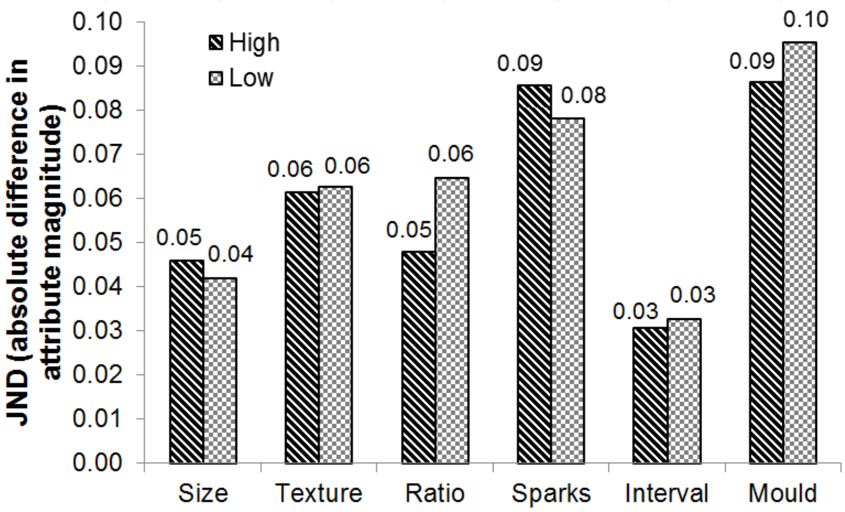
[7] Note that participants were aware that an adaptive procedure was being followed but unaware of how it worked and, hence, not able to make inferences based on the sequence of presentations. They were also aware that performance was being measured by the overall accuracy obtained, not by the proportion of correct responses. Thus, they understood that there was no gain to be had from temporarily responding incorrectly to then obtain easier trials.

[8] Twelve runs for the mould condition were excluded due to a data recording error.

[9] The statistical significance of this variability was confirmed by fitting a full generalised mixed model with logisitic link function, as described in more detail for the main experiments.

1, a similar pattern across attributes would arise.

Figure 3: Average JNDs across attribute and range condition.



4.3 Results

Mixed Effects Logits (MELs) were estimated using the following general specification:

$$In\left[\frac{\Pr(\text{“}Yes\text{”})}{1-\Pr(\text{“}Yes\text{”})}\right]_{ihcrt} = (\phi_0 + \mu_i) + (\gamma_0 + \upsilon_i)s_{ihcrt} + \boldsymbol{\phi} \boldsymbol{z}_{hrct} + \boldsymbol{\gamma} \boldsymbol{z}_{hrct} * s_{ihcrt} \tag{3}$$

where $s_{ihcrt} = \Delta_{icrt}/\beta_{h1}$ is the normalised surplus for individual, i, on trial, t, for a given hyperproduct, h, condition c and attribute range r. The fixed effects coefficients are denoted by $\phi_0, \gamma_0, \boldsymbol{\phi}$ and $\boldsymbol{\gamma}$. The model has normally distributed random effects, μ_i and υ_i with correlation $corr(\mu_i, \upsilon_i)$.[10] $\boldsymbol{z}_{hrct}$ is a vector containing the experimental manipulations of interest, including dummy variables for the relevant range, r, extent of non-linearity, $\alpha(c)$, and any other variables or interactions of potential interest. $\boldsymbol{z}_{hrct}$ enters both individually and as an interaction term with surplus, s_{ihcrt}. The vector of coefficients $\boldsymbol{\phi}$ therefore determines how bias varies across experimental conditions, while $\boldsymbol{\gamma}$ determines variation in precision.

From the properties of the logistic distribution, the average JND and PSE for a given attribute,

[10] Note that the alternative fixed effects approach, with each μ_i and υ_i included as regressors, leads to no substantive change in the overall pattern of results. Moreover, the estimated μ_i and υ_i are approximately normally distributed, supporting the assumptions of the random effects model.

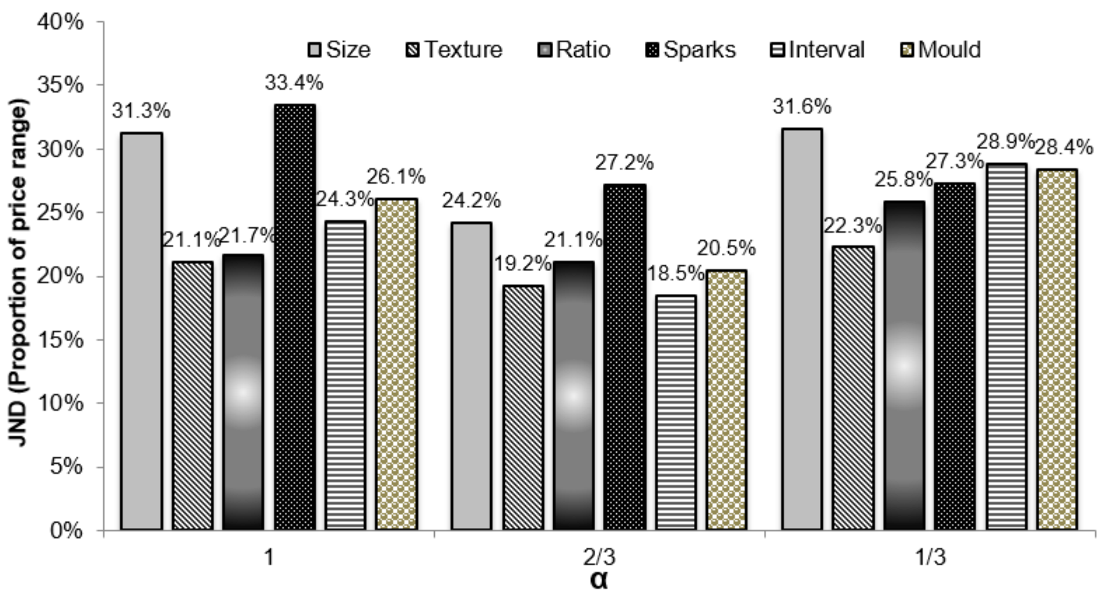
$j = 1, 2, 3, 4, 5, 6$ (two for each hyper-product), and value of $\alpha \equiv \alpha(c)$, are given by

$$JND_{j,\alpha} = \frac{\pi}{\sqrt{3}} \cdot \frac{1}{\gamma_0 + \gamma_j + \gamma_\alpha + \gamma_{j*\alpha}} \tag{4}$$

$$PSE_{j,\alpha} = -\frac{\phi_0 + \phi_j + \phi_\alpha + \phi_{j*\alpha}}{\gamma_0 + \gamma_j + \gamma_\alpha + \gamma_{j*\alpha}} \tag{5}$$

where, as in equation (3), each ϕ term is the estimated coefficient for the appropriate condition and each γ is the estimated coefficient for its interaction with the surplus. Thus, $\gamma_{j*\alpha}$ is the coefficient for the two-way interaction between the dummy variable for an attribute, the dummy variable indicating the extent of non-linearity, and the surplus. Intuitively, therefore, it estimates the impact on the slope of the psychometric function (and hence on precision) of the combination of a specific attribute and a specific α. Figure 4 presents estimated JNDs by attribute, j and condition, c.

Figure 4: Average JNDs and across α condition and attributes.



Note: JNDs estimated from MEL model with dummies for each attribute and level of α, plus interactions between the two.

Three findings are of note. The first is the level of absolute performance. Even with a single, easily perceptible attribute related monotonically to price, surplus identification was imprecise. To identify a surplus reliably, participants required it to exceed 18% of the price range, with a mean across conditions of 25.2%. Second, variation across attributes did not match that for perceptual discrimination (c.f., Figure 3), suggesting that imprecision did not result from perceptual noise. Third, the non-linearity in the attribute-price relationship had little impact. In fact, there was a slight advantage for the moderate diminishing returns case, $\alpha = \frac{2}{3}$.

Relative to imprecision, observed biases were small, The PSE was always in the region of 0-5 percentage points, albeit with a slight overall negative bias. On average, participants judged that the surplus was zero when the product was, in fact, worth 1.5 percentage points less than the displayed price.

Table 1 provides a more detailed picture. Column (1) presents an overall model, aggregated across attributes. The coefficient on the surplus is of course highly significant. The remainder of the top half of the table presents interactions that test the impact of conditions on precision. There was no overall effect of either non-linearity. The low attribute range had a modest negative effect on the precision of surplus identification, although less so in the case of moderate diminishing returns ($\alpha = \frac{2}{3}$). Precision was also influenced by location in the price range. The variable 'Z-price' corresponds to the display price expressed in standard deviations. The positive coefficient implies that participants were somewhat more precise in the upper part of the price range.

The constant and its interaction terms indicate how the bias varied across conditions. The positive coefficient confirms the slight general tendency to overestimate surplus. However, the much larger effect was how the extent of bias varied across the price range. The positive coefficient on Z-price shows that participants underestimated surpluses at the bottom of the price range and overestimated them at the top.

Columns (2)-(7) estimate the model separately for each attribute.[11] Variability in precision by non-linearity and range was unrelated to perceptual discrimination, but followed a more complex pattern. The advantage of moderate diminishing returns when the attribute range was low was driven by two attributes only: the size and texture of the Golden Egg. Improved precision at higher prices occurred for four of the six attributes. These inconsistencies may partly reflect idiosyncratic aspects of the price ranges chosen for the three hyperproducts, such as the locations of salient round prices, or may indicate non-linearities inherent in the perceptual coding of attribute magnitudes. Nevertheless, four results were consistent across attributes: (1) surplus identification was imprecise; (2) there was no advantage for linear over non-linear mappings of attributes to prices; (3) surpluses

[11]The results are presented in this way for ease of interpretation. Analysing attribute-specific effects via two-, three- and four-way interactions yields the same pattern.

Table 1: Mixed Effects Logit: Baseline models by attribute

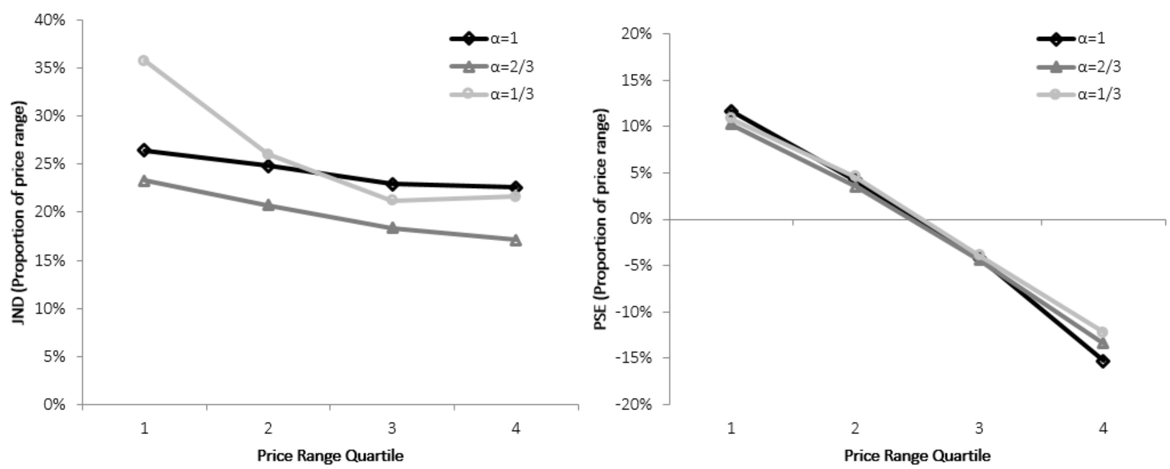
Attribute	(1) Overall	(2) Size	(3) Texture	(4) Ratio	(5) Sparks	(6) Interval	(7) Mould
Surplus	8.483***	7.574***	15.669***	9.609***	8.431***	7.167***	9.017***
	(0.754)	(1.519)	(0.952)	(1.225)	(2.099)	(1.991)	(1.067)
Surplus*α (Base=1)							
$\frac{2}{3}$	0.685	-0.578	-4.048***	-1.827	3.182	3.872	2.878
	(0.620)	(1.642)	(1.426)	(2.005)	(2.305)	(2.692)	(1.924)
$\frac{1}{3}$	0.002	-1.452	-5.056***	-0.273	1.745	3.142	0.643
	(0.935)	(1.667)	(1.278)	(1.680)	(2.474)	(3.557)	(1.623)
Surplus*Range (Base=High)							
Low	-1.919***	-2.438	-7.403***	-0.573	-5.217**	5.192**	-1.733
	(0.666)	(2.105)	(1.845)	(1.781)	(2.203)	(2.153)	(1.934)
Surplus*Range*α (Base=1)							
Low*$\frac{2}{3}$	1.390	6.241**	6.753***	2.112	-0.467	-5.375*	-2.591
	(1.001)	(2.580)	(2.248)	(2.740)	(2.752)	(2.962)	(2.689)
Low*$\frac{1}{3}$	-0.200	2.505	4.767**	-1.626	1.163	-8.525**	-2.084
	(1.073)	(2.532)	(2.139)	(2.349)	(3.066)	(4.345)	(2.643)
Surplus*Z-price	0.846***	1.567***	1.332**	1.312*	0.383	1.266**	-0.109
	(0.209)	(0.362)	(0.605)	(0.722)	(0.584)	(0.623)	(0.667)
Constant	0.258***	0.267	0.598*	0.678***	0.014	0.452*	-0.160
	(0.095)	(0.188)	(0.306)	(0.161)	(0.180)	(0.252)	(0.265)
α (Base=1)							
$\frac{2}{3}$	-0.153	0.029	-0.596	-0.861***	0.072	-0.114	0.280
	(0.104)	(0.272)	(0.458)	(0.180)	(0.259)	(0.337)	(0.345)
$\frac{1}{3}$	-0.072	0.211	-0.253	-0.253	0.046	-0.718**	0.311
	(0.108)	(0.345)	(0.513)	(0.186)	(0.405)	(0.301)	(0.318)
Range (Base=High)							
Low	-0.280*	0.093	-0.202	-0.264	-0.412*	-0.832**	-0.225
	(0.145)	(0.222)	(0.344)	(0.203)	(0.224)	(0.387)	(0.389)
Range*α (Base=1)							
Low*$\frac{2}{3}$	0.256	-0.016	0.793	-0.261	0.287	0.981**	0.236
	(0.194)	(0.387)	(0.572)	(0.262)	(0.448)	(0.444)	(0.460)
Low*$\frac{1}{3}$	0.317**	0.085	0.316	0.590*	0.315	0.569	0.205
	(0.145)	(0.344)	(0.506)	(0.348)	(0.391)	(0.486)	(0.475)
Z-price	0.733***	0.565***	1.208***	0.632***	0.769***	0.832***	0.760***
	(0.064)	(0.098)	(0.104)	(0.106)	(0.101)	(0.125)	(0.109)
Random effects parameters							
Var(μ_i)	5.019***	5.602***	2.554*	4.930***	7.128***	15.372***	4.529*
	(1.023)	(2.088)	(1.385)	(1.770)	(2.342)	(4.753)	(2.439)
Var(υ_i)	0.052***	0.116*	0.305**	0.060	0.194***	0.183**	0.189***
	(0.016)	(0.064)	(0.140)	(0.059)	(0.071)	(0.072)	(0.053)
Cov(μ_i, υ_i)	-0.069	-0.101	0.357	-0.354**	-0.204	-0.175	0.521*
	(0.095)	(0.218)	(0.222)	(0.144)	(0.263)	(0.386)	(0.317)
Observations	15,552	2,592	2,592	2,592	2,592	2,592	2,592
Number of groups	36	36	36	36	36	36	36

Robust standard errors in parentheses. Clustered at the individual level. *** p<0.01; ** p<0.05; * p<0.1

were underestimated for low-value products and overestimated for high-value ones; (4) there was heterogeneity across individuals, with the standard deviation across consumers equivalent to a difference in JND of approximately 6 percentage points of the price range.[12]

Figure 5 provides more intuition by presenting the estimated JND and PSE across price quartiles, split by α condition. This confirms the modest improvement in precision at higher prices. The JND for the first price quartile with severe diminishing returns ($\alpha = \frac{1}{3}$) stands out, suggesting that when small changes in attribute magnitude translated into large changes in price, perceptual constraints eventually reduced precision. However, changes in precision across the price range were small compared to changes in bias, which were approximately linear and consistent across conditions.

Figure 5: JND and PSE by α across price range



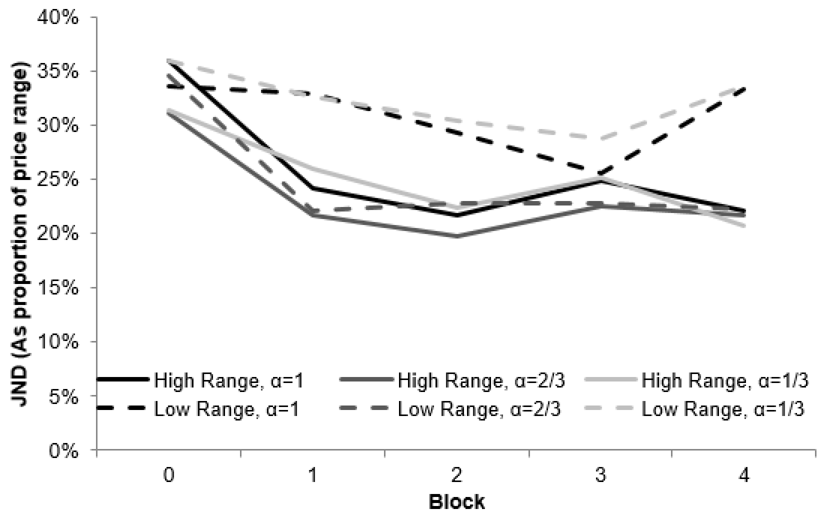
Note: The above estimates of the PSEs and JNDs are based on an estimated MEL that allows for all possible two- and three- way interactions between the price range quartiles, α conditions and the size of the surplus.

4.3.1 Learning

Trial number within each experimental run and its squared term were added to the baseline specification. Perhaps surprisingly, neither had a statistically significant impact on precision or bias (p>0.25), indicating a lack of learning despite multiple exposures with feedback. To examine this

[12] This is captured by the random effects parameter var(μ_i). While there was also variation in bias across individuals, its magnitude was small.

Figure 6: Average JND over time per experimental run.



Note: Each PSE is computed based on the parameter estimates from an MEL with non-linearity, range and block dummies specified, as well as their interactions. Block 0 refers to the initial 8 practice trials participants completed.

further, dummies separating each experimental run into four blocks of 18 trials were added to the specification, which was also expanded to include a fifth block for the initial (non-incentivised) practice trials (block 0). There was a statistically significant improvement in precision between the practice and first block of test trials ($p<0.001$), but not thereafter. Although this difference may reflect factors other than learning, such as participants experimenting with different strategies, the implication remains that any learning was rapid. Figure 6 shows how JNDs evolved across experimental runs for each α and range condition. For four of the six conditions, peak performance was achieved after the practice trials, while for the other two learning was not robust.

A small learning effect did emerge over the whole experimental session ($p<0.1$), equivalent to a decrease in JND of approximately three percentage points after 300 trials. This effect size is small and its positive direction suggests that fatigue was not a factor. There was no equivalent improvement in bias.

4.4 Discussion

The S-ID task produces quantitative measures of precision and bias in surplus identification. Facing a novel product with a single, readily perceptible visual attribute, surplus identification is imprecise and systematically biased over the price range. Performance is clearly constrained by some form of cognitive limitation.

The simplicity of the economic decision in Experiment 1 is important. Several recent models that impose cognitive constraints on consumer choice hinge on how agents allocate limited attention (Lipman, 1995; Sims, 2003), focus on a subset of relevant information (Bordalo et al., 2013; Kőszegi and Szeidl, 2013) or how they account for inevitable perceptual error (Woodford, 2012; Caplin and Martin, 2015). Participants in Experiment 1 devoted their full attention to one salient attribute, so allocation of attention across attributes was not involved. Similarly, attributes were designed to minimise perceptual error. Indeed, the pilot study showed that relative magnitudes could be perceived with high but variable precision across attributes. Contrastingly, surplus identification was consistently imprecise. The limiting factor was, plainly, not perceptual in the sense of discriminating relative magnitudes. Nor was performance affected by non-linear returns to attributes, despite previous evidence that in some contexts consumers fail to account for non-linearities (see Section 2).

Instead, the imprecision of surplus identification in Experiment 1 implies a more fundamental limitation, the locus of which is neither perceptual, nor attentional, nor related to the shape of returns. Rather, the ability to identify surpluses is limited by the need to compare relative location on two otherwise incommensurate internal scales, one for monetary amounts, the other for attribute magnitudes. The results suggest that internal representations of attributes can vary in granularity (i.e., perceptual error) and linearity with only minimal impact on performance. Precision of surplus identification is instead dominated by the mapping of one internal scale on to another, with limited scope for learning once an initial mapping has been established. Such a constraint would explain the consistent absolute level of performance, whereby the average observer requires a surplus equivalent to one fifth of the price (or, equivalently, attribute) range for detection to be reliable.

An advantage of the S-ID task is that it generates separate measures of precision and bias. While constraints in the mapping of incommensurate internal scales might explain uniform imprecision across attributes, the consistent bias across the price range also requires explanation. Another way to describe this bias is that, over trials, participants responded more to variation in attribute magnitudes than to variation in prices. However, since this effect was independent of attribute, linearity of returns and price range (which differed across products by orders of magnitude), it might reflect a tendency for variation on a perceptual scale to be overestimated in comparison with variation on a numeric one. This could happen if, for instance, the perceptual scale were subject to visual contrast effects, leading differences in attribute magnitudes between successive presentations to be exaggerated.

5 Experiment 2

The negligible impact of non-linear returns on surplus identification in Experiment 1 invites further increases in the complexity of returns to attributes. The attribute-price relationships of everyday products are frequently more elaborate than simple monotonic functions. What forms of complexity might disrupt the uniformity of precision?

Experiment 2 increased the complexity, first, by incorporating a second attribute and, second, by defining the attribute-price relationship via a range of more complex and economically interesting functional forms, including some standard preference functions. (Hereafter we refer to the function defining the attribute-price relationship as the "value function".) These included functions with increasing returns as well as non-monotonic and cyclical relationships.

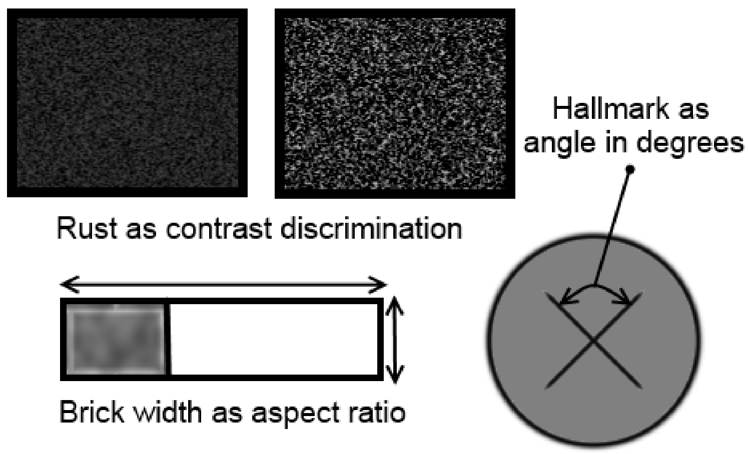
The multi-dimensional attribute space also allowed us to test additional hypotheses arising from Experiment 1. First, we reasoned that if biases when mapping attributes to prices result from differences between the internal representations of visual and numeric quantities, a numeric product feature would alter the bias. In half of the experimental runs, therefore, the magnitude of one of the two attributes was presented numerically. Second, the use of numeric attribute magnitudes would eliminate any error due to visual perception. Third, since adding a second attribute introduces a new source of complexity in the form of the relative attribute weighting, we tested whether performance was affected by relative weight. Lastly, and most straightforwardly, we hypothesised that learning would be slower with more complex two-attribute products.

5.1 Method

Consumers from the Dublin area (N=24) were recruited through a market research company, with approximate balance by gender (14 female), age (M=34.7, SD=13.0), and occupational status (54% employed). Methods were as in Experiment 1, except for the following modifications.

First, six additional attributes were employed.[13] Three, one for each hyperproduct, were visual (see Figure 7). On the Golden Egg, we varied a quality hallmark, the magnitude of which was defined as the angle subtended by two intersecting lines. On the Victorian Lantern, we varied the "rustiness" of the metal, defined as the contrast of an orange-brown versus black coloured texture. On the Mayan Pyramid, we varied the flatness of the bricks, defined as the rectangular aspect ratio. These visual stimuli were selected on the basis of the human ability to discriminate angle, contrast and shape with relatively high precision.

Figure 7: Additional continuous attributes in Experiment 2



The other three attributes, again one for each hyperproduct, were numeric and appeared on a label next to each hyperproduct (see figure 8). For the Golden Egg, we displayed the purity in carats on a plinth; for the Victorian Lantern, fuel efficiency on a 25-point gradient scale; for the Mayan Pyramid, age in years on a scroll.

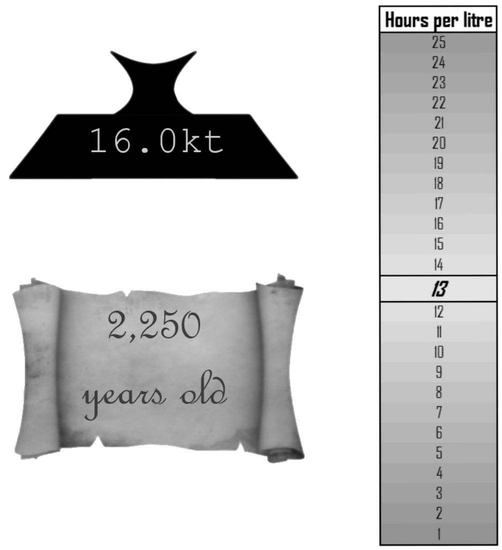
Extending equation (2) to the two-attribute case,

$$P_{htv} = \beta_{h0} + \beta_{h1} f_v(x_{1t}, \alpha_1, x_{2t}, \alpha_2) \qquad f_v(\cdot) \in [0,1] \qquad \alpha_1 + \alpha_2 = 1 \tag{6}$$

where v denotes one of six value functions and the βs map the overall product magnitude, $f_v(\cdot)$, onto the price range for a given hyperproduct, h. Table 2 shows the six functional forms of $f_v(\cdot)$, which were designed to increase the complexity of the attribute-price relationship. Function (1) was linear, with perfectly separable attributes. Function (2) had constant returns to scale overall, but

[13] Again, precise detail of the generation of images is available in supplementary material.

Figure 8: Numeric attributes in Experiment 2



diminishing returns (DRS) per attribute. Hence, these two value functions were equivalent to those in Experiment 1, although with two-dimensions they differed with regard to separability. Function (3) exhibited increasing returns (IRS) overall and for at least one attribute. Function (4) consisted of a standard preference function in which attributes were perfect complements: the product was as good as its weakest attribute (Leontief preferences). This specific form of complexity required participants, first, to make a relative comparison of the attribute magnitudes and, then, to compare the weakest against the displayed price. Function (5) combined one attribute with linear returns with a more complex non-linear periodic attribute. We hypothesised that a cyclical attribute, with more complex non-linear returns, would reduce precision and slow learning. Finally, function (6) applied a non-monotonic non-linearity to both attributes, such that the centre of the attribute space defined a perfect product. We called this the"goldilocks" value function, because the product price corresponded to the distance in attribute space from the "just right" attribute levels.

Participants completed one run per value function. Each began with a learning phase in which systematic examples of hyperproducts and prices were shown. In Figure 9, the order and locations of the examples in attribute space are shown, together with indifference curves assuming balanced attribute weights. Participants then undertook eight practice trials and 56 test trials (t). After the product price, P_{htz}, was drawn (identically to Experiment 1), one combination of x_{1t} and x_{2t} was selected at random to match it. While α_1 and α_2 always summed to one, they were balanced

Table 2: Value function specifications

(1)	Perfect Substitutes (linear)	$\alpha_1 x_1 + \alpha_2 x_2$
(2)	Cobb–Douglas (constant returns to scale)	$x_1^{\alpha_1} x_2^{\alpha_2}$
(3)	Cobb–Douglas (increasing returns to scale)	$x_1^{3\alpha_1} x_2^{3\alpha_2}$
(4)	Leontief	$max(\alpha_1, \alpha_2) min\left(\frac{x1}{\alpha_1}, \frac{x_2}{\alpha_2}\right)$
(5)	Cyclical	$\alpha_1 x_1 + \alpha_2 \frac{sin(2\pi x_2)+1}{2}$
(6)	Goldilocks	$1 - \frac{4}{\frac{1}{\alpha_1^2}+\frac{1}{\alpha_2^2}} \left[\left(\frac{x_1-\frac{1}{2}}{\alpha_1}\right)^2 + \left(\frac{x_2-\frac{1}{2}}{\alpha_2}\right)^2\right]$

(i.e., equally weighted, $\frac{1}{2}$,$\frac{1}{2}$; $b = 1$) for half the runs and unbalanced ($\frac{2}{3}$,$\frac{1}{3}$ or $\frac{1}{3}$,$\frac{2}{3}$; $b = 2$) for the other half. Value functions, products, combinations of attribute pairs, and attribute balance were all pseudo-randomised across participants and runs.

5.2 Results

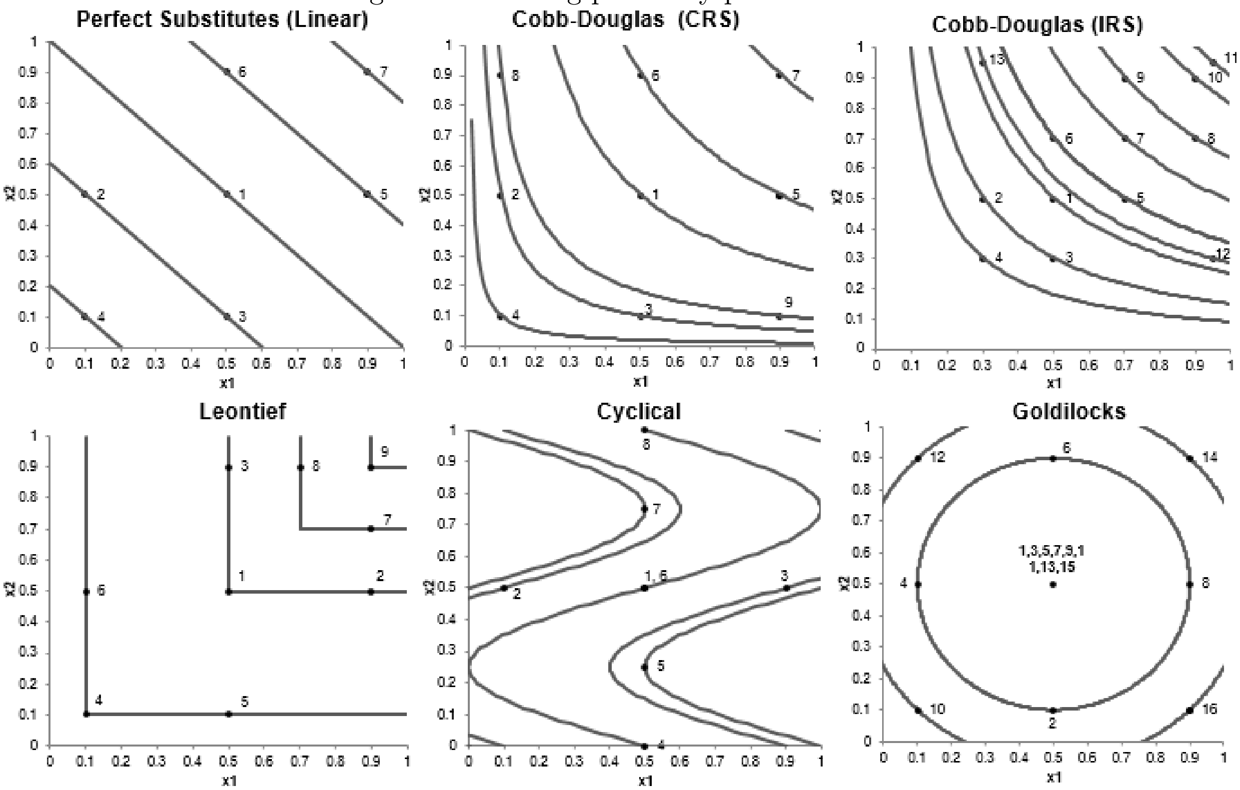
MEL models were estimated following the specification in equation 1, with only the exogenous variables $\boldsymbol{z}_{htvb}$ augmented. The baseline model included dummy variables for value function, v, whether attributes were balanced, b, and interactions between the two.[14] The results (Table 2, column (1) and Figure 10) reveal that increasing the complexity of the value function disrupted surplus identification.

Before examining this variation more closely, the absolute level of performance invites comment. In Experiment 1, a single attribute range matched the full price range. In Experiment 2, on average, the range of each attribute mapped on to only half the price range. The JNDs in Figure 10 are measured, therefore, as a proportion of an attribute range, such that precision when an attribute was mapped to price on its own can be compared directly to precision when a second attribute was simultaneously taken into account. In Experiment 2, reliable surplus identification with the easiest value functions required a surplus equivalent to one third to one half of an attribute range, compared to one fifth to one quarter in Experiment 1 (Figure 4). The higher JNDs indicate substantial loss of precision when the second attribute had to be considered simultaneously.

While overall precision was reduced, Figure 10 again reveals little difference between monotonic

[14] As for Experiment 1, estimating a model with fixed effects for individuals revealed them to be approximately normally distributed, supporting the more parsimonious random effects model.

Figure 9: Learning phases by price function



linear and non-linear value functions. The Leontief value function with balanced attributes was the only condition to produce JNDs as low as Experiment 1. The more complex periodic and goldilocks value functions resulted in substantially greater imprecision. Table 2, column (1) provides significance tests. The slight deterioration in precision with increasing returns was marginally outside conventional levels of statistical significance, while the improvement in the Leontief case was significant only when attributes were balanced. However, the deterioration with non-monotonic value functions was highly statistically significant.

Although the complexity of the value function had a strong impact, variation in the relative weights of the attributes did not. Unbalanced attributes significantly disrupted the Leontief value function only. The additional complexity introduced by this value function primarily surrounded the need for participants to make two sequential judgements, first assessing relative attribute magnitudes, then the relationship of the weakest attribute to price. It is likely that the first stage was disrupted by unbalancing the attributes.

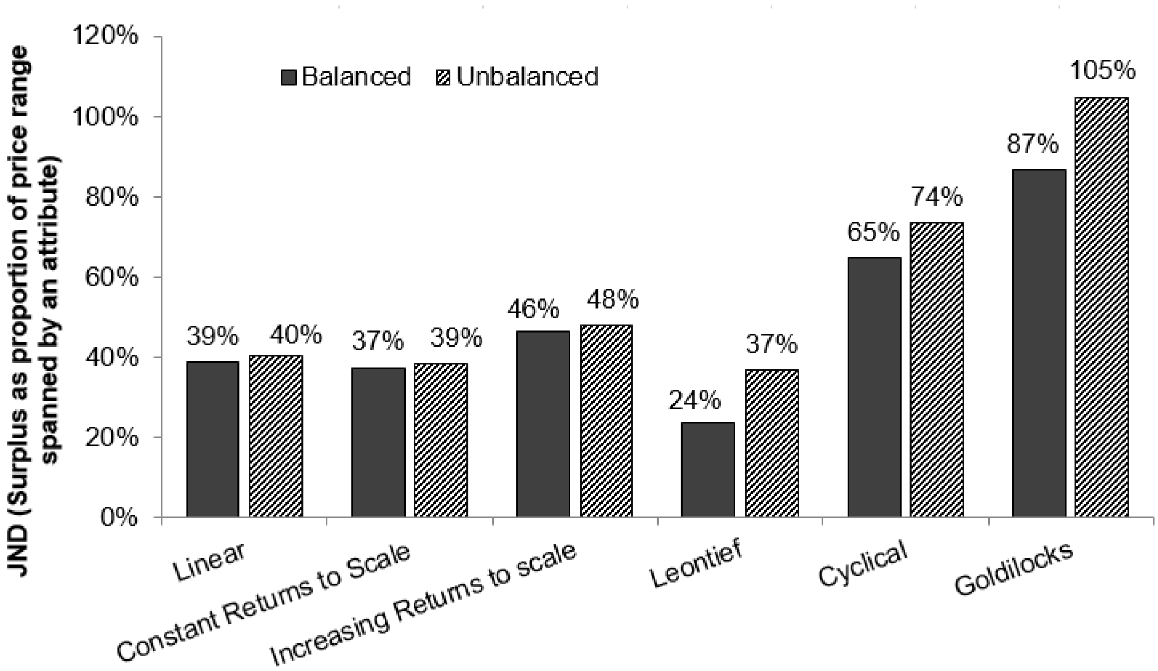
Surplus identification was again biased, with poorer products undervalued and better products

Table 3: Mixed Effects Logits: Testing for variable in ability across value function

	(1)	(2)	(3)	(4)
Surplus	4.668***	4.277***	4.796***	4.361***
	(0.437)	(0.501)	(0.466)	(0.525)
Surplus*Value Function (Base=Linear)				
Constant Returns to Scale	0.171	0.413	0.218	0.499
	(0.535)	(0.574)	(0.538)	(0.563)
Increasing Returns to scale	-0.756*	-0.643*	-0.787*	-0.651*
	(0.434)	(0.380)	(0.447)	(0.381)
Leontief	3.000***	3.153***	3.393***	3.567***
	(0.502)	(0.428)	(0.560)	(0.476)
Cyclical	-1.862***	-1.820***	-1.876***	-1.828***
	(0.436)	(0.430)	(0.448)	(0.446)
Goldilocks	-2.577***	-2.440***	-2.671***	-2.517***
	(0.380)	(0.398)	(0.355)	(0.380)
Surplus*Unbalanced Attributes	-0.171	-0.003	-0.225	-0.021
	(0.451)	(0.418)	(0.473)	(0.427)
Surplus*Unbalanced Attributes*Value Function (Base=Linear)				
Lenotief	-2.584***	-2.867***	-2.955***	-3.272***
	(0.818)	(0.787)	(0.883)	(0.839)
Surplus*Numeric attribute		0.583***		0.647***
		(0.224)		(0.237)
Surplus*Z-price			0.084	0.183
			(0.145)	(0.198)
Surplus*Numeric attribute*Z-price				-0.208
				(0.252)
Constant	0.311	0.267	0.298	0.242
	(0.217)	(0.222)	(0.223)	(0.225)
Value Function (Base=Linear)				
Constant Returns to Scale	-0.634**	-0.613**	-0.624**	-0.598**
	(0.263)	(0.258)	(0.273)	(0.266)
Increasing Returns to scale	-0.197	-0.186	-0.169	-0.154
	(0.396)	(0.388)	(0.413)	(0.404)
Leontief	-0.279	-0.269	-0.289	-0.274
	(0.307)	(0.303)	(0.323)	(0.317)
Cyclical	-0.407*	-0.405*	-0.387	-0.380
	(0.233)	(0.231)	(0.244)	(0.246)
Goldilocks	-0.670***	-0.663***	-0.695***	-0.684***
	(0.255)	(0.253)	(0.262)	(0.260)
Unbalanced Attributes	-0.407	-0.396	-0.408	-0.390
	(0.266)	(0.261)	(0.279)	(0.272)
Numeric attribute		0.072		0.091
		(0.103)		(0.106)
Z-price			0.490***	0.443***
			(0.053)	(0.062)
Numeric attribute*Z-price				0.098
				(0.091)
Random effects parameters				
Var(μ_i)	1.670***	1.694***	1.834***	1.865***
	(0.419)	(0.425)	(0.451)	(0.456)
Var(υ_i)	0.011*	0.011*	0.016*	0.016*
	(0.006)	(0.006)	(0.009)	(0.009)
Cov(μ_i,υ_i)	-0.011	-0.010	-0.032	-0.029
	(0.051)	(0.052)	(0.060)	(0.060)
Observations	8,064	8,064	8,064	8,064
Number of groups	24	24	24	24

Robust standard errors in parentheses. Clustered at the individual level. *** p<0.01; ** p<0.05; * p<0.1

Figure 10: JNDs across value function and attribute balance conditions



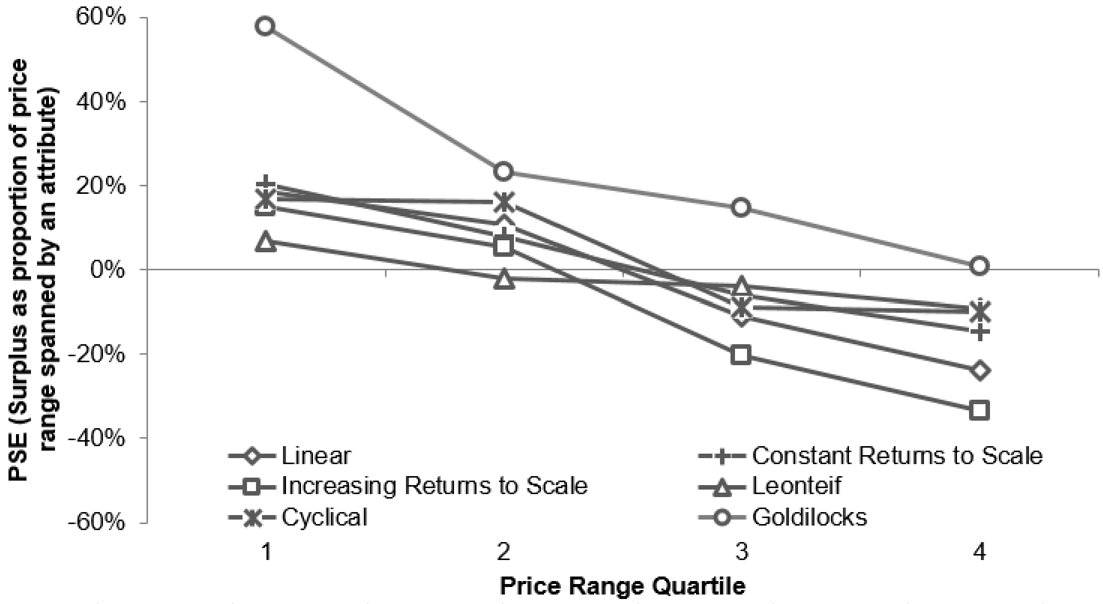
Note: Each JND was computed based on a MEL that included the magnitude of the surplus, value function dummies, and an unbalanced weight dummy, as well as all two-way and three-way interaction terms.

overvalued. The consistency of this effect across value functions is evident from Figure 11 and its statistical significance from Columns (2)-(4) of Table 2. Contrary to our hypothesis that this bias might be caused by visual contrast effects, the bias was somewhat larger in the presence of a numeric attribute, as indicated by the positive interaction between numeric attributes and the normalised display price (*Z-price*). The numeric attribute did generate a slight but significant improvement in precision. Translating the coefficients from Table 2, when the visual attribute was replaced by a numeric one, for a monotonic value function, the JND fell from approximately 42% to 37% of the attribute range.

5.2.1 Learning

It is important to investigate whether deterioration in the precision of surplus identification reflected cognitive limitations or slower learning. As in Experiment 1, adding the trial number and its squared term to the baseline model yielded no effect. Including the practice trials in the estimation produced a significant positive coefficient ($p<0.05$) on the trial number and a significant negative coefficient

Figure 11: Variation in the PSE across the price range.



Note: Each PSE has been computed based on the parameter estimates from an MEL with all value function dummies, price quartile dummies, and their interactions included.

($p<0.05$) on its square, in keeping with a rapid asymptotic learning curve. This pattern was not consistent across value functions, however. Performance actually deteriorated for the goldilocks value function relative to the practice trials ($p<0.05$). Indeed, this effect was large: average JND was 32.2% percentage points higher than during the practice trials. The need to make judgements relative to an absolute benchmark, a "just right" point retained in memory, appears to have greatly increased imprecision despite repetition and feedback.

As in Experiment 1, the bias across the price range did not diminish; in fact it strengthened marginally over the session. In contrast to Experiment 1, there was no overall improvement across the experimental session.

5.3 Discussion

Experiment 2 confirms and extends the findings of Experiment 1. In this simplified and experimentally controlled environment, with training and feedback over many trials, surplus identification is imprecise, biased and subject to only modest learning. The absence of an advantage for linear over non-linear (monotonic) returns applies for multiple (two) attributes. Once the complexity of the non-linearity is increased to include turning points, however, performance declines substantially. Attributes that can be both too large and too small are common. Examples include portion sizes for food and drink, terms of loans, engine sizes; consumers often seek a "happy medium".

The systematic bias across the price range is unaffected by the use of a numeric attribute, which suggests it does not result from contrast effects associated with visual attributes. We return to alternative possible causes in the General Discussion. Numeric attributes can, it seems, make a marginal improvement to the precision of surplus identification, suggesting perhaps a small role for perceptual error, but with the dominant capacity constraint in surplus identification being the need to map one internal scale on to another.

6 Additional Tests and Analyses

This section details additional tests and analyses designed to address two issues regarding the generalisability of our results. First, surplus identification may require learning over days rather than an hour, especially for complex non-linear value functions, where lack of learning could reflect poor understanding rather than limited information integration. To explore this possibility, we recruited some highly numerate economics students to compete in a surplus identification tournament spanning more than a week. Second, one could of course question whether our results generalise to subjective choices among more familiar products. The S-ID task imposes preferences upon participants via incentives to match a predetermined function. It is not certain that this process engages the same evaluation mechanisms as the identification of purely subjective surpluses. However, the richness of the S-ID task data allow some instructive additional tests. Specifically, we test whether several biases repeatedly documented in subjective consumer choice tasks also appear in our data. In addition, we test specific predictions of Woodford's (2014) Optimal Sensor Model, which have been confirmed in subjective choice data. Positive results, in these tests strengthen the case that common psychological mechanisms underpin responses in the S-ID task and subjective consumer choices.

6.1 Additional test: send in the experts

Employing the experimental design of Experiment 2, a small group of economics undergraduates and postgraduates (N=6) undertook four repeated sessions, separated by at least 48 hours. All were highly numerate and understood iso-value curves. Prior to each run, they were shown the exact functional form of $f_v(.)$, including graphical examples. Thus, any risk of misundersting was eliminated. To increase opportunities for learning, value functions, attributes and weightings for each participant remained the same across sessions; only presentation order varied. The experiment was a tournament: the best performer won €50. Furthermore, participants were told before the third session that they would win €5 if performance in their third or fourth session improved on their first or second session respectively.

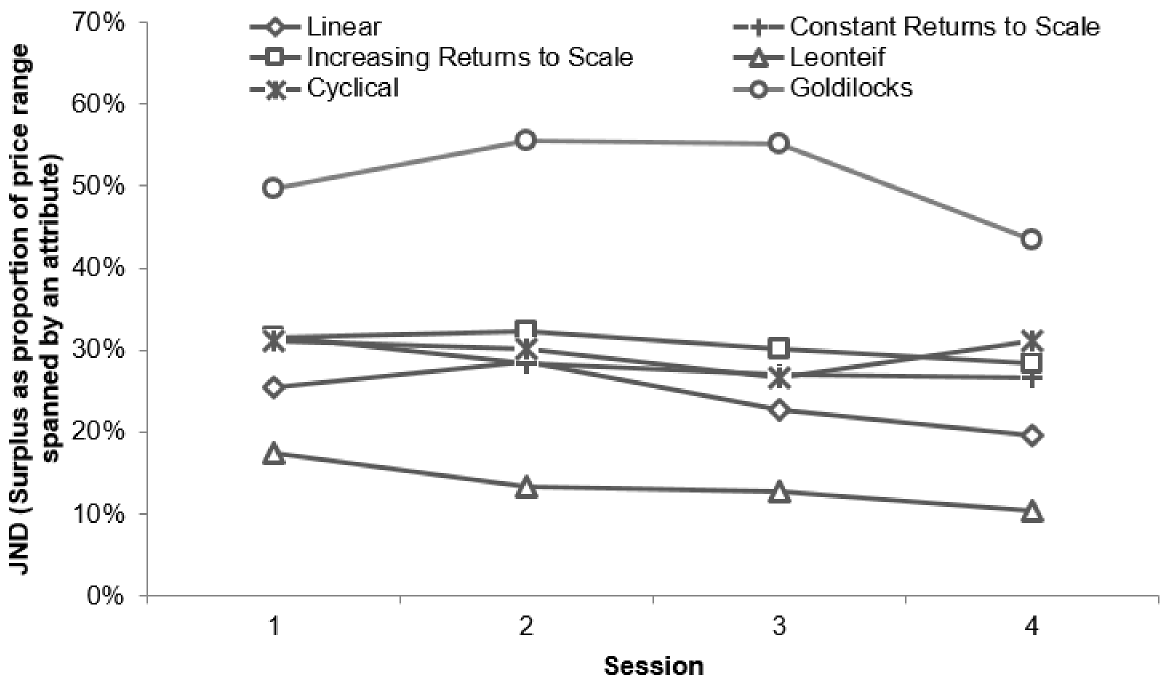
Figure 12 presents mean JNDs by value function across sessions. Surplus identification remained imprecise, but JNDs were lower than for the sample of Dublin consumers, confirming this cohort's "expert" status.[15] Reliable identification in the goldilocks case still required a surplus equivalent to half an entire attribute range, but precision for the periodic function approached that for the monotonic functions. The main purpose of this additional test was to provide greater opportunities for learning, yet it remained modest. Over four sessions totalling 1,536 trials with feedback, competing for meaningful rewards, there was a slight improvement in precision for the monotonic value functions ($p<0.05$), but the effect was very small. Meanwhile, the large and systematic bias across the price range was replicated ($p<0.001$) and immune to learning. Thus, while young, highly numerate students can outperform a representative sample of consumers, imprecision and bias are not easily overcome. The only outstanding query from this exercise is the better relative precision with the periodic value function. While we cannot be sure, participants seemed to find the graphical presentation particularly helpful, perhaps allowing them to match turning points to prices more easily.

6.2 Tests for biases in common with subjective choice tasks

The S-ID task mimics the situation where a consumer encounters a new product and begins to learn its worth. Yet the cost of gaining complete scientific control over attributes, prices and surpluses is the need to impose preferences. If the S-ID task data were to contain similar patterns to those seen in subjective consumer choice experiments, however, this would support the contention of common psychological mechanisms. We therefore tested our data for some specific biases previously observed in choice experiments.

[15] The average percentile rank for the experts across the 6 value functions and four sessions was the 93^{rd} percentile, relative to the consumer sample.

Figure 12: JNDs across sessions for our expert sample



Note: Each JND has been computed based on the parameter estimates from an MEL with all value function dummies, session number dummies, and their interactions included.

6.2.1 Dilution and familiarity effects

Consumers often struggle to ignore irrelevant attributes. Meyvis and Janiszewski (2002) call this the "dilution" effect and demonstrate across ten studies that irrelevant information alters consumer product appraisals. The data from Experiment 1 are ideal for testing for dilution, because participants focused on one attribute while an irrelevant one varied randomly. Half the time, the attribute to be ignored had been relevant on a previous run. We therefore added to the baseline MEL model (Table 1, column 1) variables for the price signalled by the irrelevant attribute, for whether it had been previously relevant and for the interaction between the two. Consistent with dilution, the higher the value signalled by the irrelevant attribute, the greater the probability that the participant perceived a surplus ($p<0.01$).[16] Interestingly, the interaction term was non-significant; the effect did not depend on whether the attribute had previously been relevant. This may reflect the design of the hyperproducts, because attributes were selected to make intuitive sense, e.g., bigger

[16]It is important to note that because our method permits the separation of bias and precision in surplus identification, the presence of a bias such as this does not imply that we overestimated JNDs or, equivalently, underestimated precision.

eggs were more valuable, rusty lanterns less, etc.

6.2.2 The attraction effect

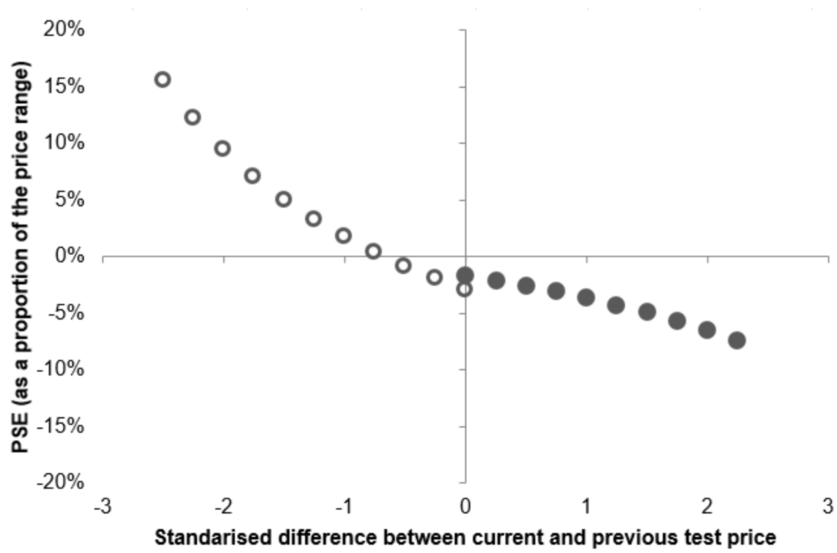
The "attraction effect" refers to the greater likelihood of choosing a product that dominates another on all attributes (Huber et al., 1982) and has been demonstrated for both matching and choice tasks (Tversky et al., 1988). Although in our experiments only one product was visible during any one trial, trials were sequential and the relationship between the attribute magnitudes of successive products was essentially random. Hence, in Experiment 2, for some trials the product was better on both attributes than the previous one, thereby dominating it. Conversely, for some others it was dominated. We tested whether participants were biased by domination relative to the previous product, by adding two dummy variables to the baseline model. There was indeed an attraction effect: domination of the previous product exaggerated perceived surplus ($p<0.05$), while being dominated by it diminished perceived surplus ($p<0.1$).

6.2.3 Loss aversion

Loss aversion was first formalised and investigated empirically by Kahneman and Tversky (1979). An extensive literature has explored asymmetries in how humans and animals weight losses and gains in choices. While explanations for these empirical phenomena remain controversial, replicable findings that imply loss aversion in economic choice are numerous (Rick, 2011; Ericson and Fuster, 2014).

Each successive presentation in the S-ID task entails an increase or decrease in attribute magnitude relative to the previous product (and feedback price). To the extent that the most recently perceived attribute provides a reference point, loss aversion implies a downward bias in perceived surplus on trials when the attribute magnitude decreased, compared to those when it increased, all else equal. Experiment 1 offers an ideal test, as successive presentations varied in a single monotonic attribute. We added to the baseline model a variable for the standardised change in attribute magnitude relative to the previous product, interacted with a dummy to indicate a decrease or an increase. This revealed a highly significant contrast effect: the difference in successive magnitudes was exaggerated ($p<0.001$). Furthermore, there was a significant interaction with the dummy variable($p<0.05$), indicating asymmetry between decreases and increases. The impact on the PSE is shown in Figure 13. The contrast effect for decreases in attribute magnitude was slightly more than twice that associated with equivalent increases, as is typically observed in studies of loss aversion in choice experiments.[17]

Figure 13: PSEs for changes in price relative to previous product.



Note: PSE was computed from parameter estimates from an MEL model including the standardised difference in attribute magnitude between the current and previous product, interacted with a dummy variable indicating whether the difference was positive (gain) or negative (loss). Interaction terms and standardised display price (Z-price) were included as controls.

6.3 Testing predictions of Woodford (2014)

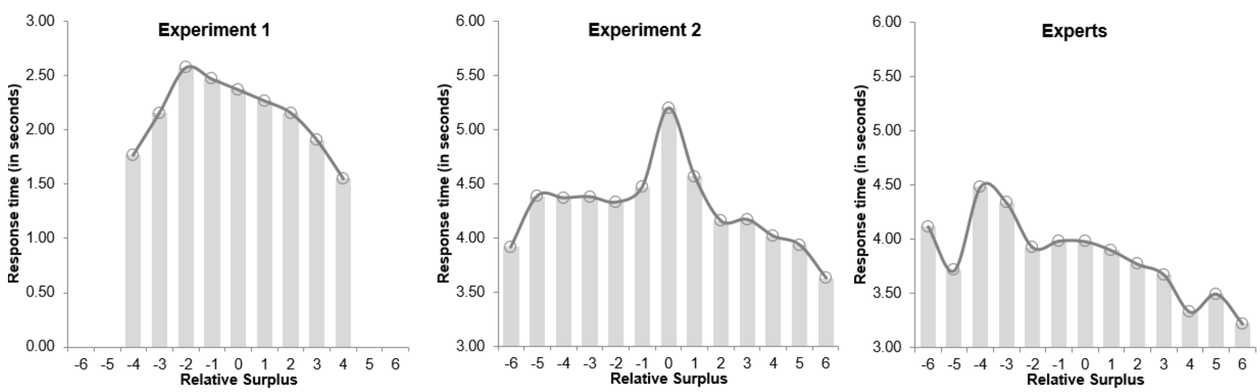
Woodford (2014) demonstrates that his Optimal Information Constrained Model (OICM) of discrete choice predicts patterns of response times when individuals choose between food items (Krajbich et al., 2010). Although our experiments were not designed explicitly to test this model, there are two reasons to check for consistency with the OICM. First, consistency would support a broader application of the OICM model. Second, finding a similar pattern of response times in the S-ID task would again suggest common mechanisms underpinning responses in the S-ID task and subjective consumer choice.

[17]The standardised price range variable, Z-price, was included in all specifications to control for potential confounds between these two simultaneously observed biases. We also tested higher powers both of Z-price and of the difference in attribute magnitude between successive presentations. While the inclusion of these covariates somewhat altered the estimated effect size and linearity of the bias, all specifications produced a steeper effect for decreases than for increases

Building on the "drift diffusion" models of mathematical psychology (e.g., Shadlen et al. (2006)) the OICM predicts: (1) that smaller surpluses should be harder to identify; (2) that more difficult decisions should induce longer response times, as neural processes require longer to accumulate evidence in support of one of the two alternatives; (3) that response times should be longer for incorrect than for correct choices. The third prediction is key for the OICM, because it derives from its central proposition of optimal accumulation from a series of signals. Given constrained information processing capacity, an optimal system updates the probability of receiving signals according to the history of signals received so far, such that (probabilistically less likely) incorrect signals take longer to accumulate.

The data from Experiment 1 and Experiment 2 confirm these predictions. Smaller surpluses induced more errors ($p<0.001$). The average response time with single-attribute monotonic functions in Experiment 1 was 2.24 seconds, while for the more difficult two-attribute monotonic functions in Experiment 2, it almost doubled to 4.02 seconds. For still more difficult non-monotonic functions it was 4.58. For the "expert" group, who had higher levels of precision, equivalent times were 3.61 seconds for monotonic and 3.94 seconds for non-monotonic. Figure 14 shows response times from both experiments by absolute magnitude of the difference between P^d_{ht} and P_{ht}, where positive values indicate correct and negative values incorrect decisions. Response times were longer for more smaller surpluses and, crucially, for incorrect decisions.

Figure 14: Response times by relative surplus mangitude



Note: Positive values indicate that surplus was correctly identified and negative ones indicate incorrect choice.

7 General Discussion

Surplus identification is subject to important capacity constraints even when just one or two directly observable attributes are compared with a price. Experiment 1 shows that for simple, monotonic one-to-one mappings from attribute to price, surpluses equivalent to 18-34% of the attribute or (equivalently) price range are required for reliable detection. Experiment 2 shows that the addition of a second attribute causes the surplus required to climb to 37-48%.[18] Thus, consumers can map incommensurate attributes and prices onto common scales only with substantial imprecision. Moreover, while surplus identification is not disrupted by monotonic non-linearities, non-monotonic returns produce a further large decrease in precision. In addition to these bottlenecks, both experiments reveal a systematic and persistent bias, whereby better products are overvalued and worse ones undervalued.

The generalisability of these results is, of course, open to debate. Repeated identification of objective monetary surpluses may not involve precisely the same mechanisms as sporadic assessment of the subjective benefits of purchases. Nevertheless, common mechanisms are implied by the existence of biases within the S-ID task previously demonstrated in choice experiments. Furthermore, the findings are consistent with previous evidence on relative versus absolute perception. Our JNDs suggest that consumers can reliably map just four-to-seven levels of a single attribute on to a price range. Similarly, performance in "absolute identification" experiments, in which participants must identify which of a set of stimuli of ascending magnitude is presented, deteriorates once the number of stimuli in the set approaches seven or more (Laming, 1997; Stewart et al., 2005), even though individuals can perform much more accurate relative discriminations between simultaneously presented stimuli.

The overall pattern of errors in surplus detection uncovered by our experiments has implications for the theoretical development of choice models. At the most general level, it suggests that the inclusion information-processing constraints is well-founded. Multi-attribute economic choice appears prone to substantial errors and an accurate descriptive theory should be consistent at least with large, systematic ones. Because the S-ID task locates and measures errors in surplus identification in an experimentally controlled fashion, it can serve as an experimental complement

[18] In related work, Lunn and Bohacek (2015) find that adding a second, third and fourth attribute reduces precision by more than is consistent with statistically efficient integration of single-attribute judgements.

to choice models that incorporate bounded cognition. Here we consider four specific issues raised.

Firstly, the results have implications for the cause of imprecision. The models reviewed in Section 2.2 mostly centre on the allocation of attention across multiple sources, resulting in inaccurate perception or weighting. That is, the agent's problem is *excess* information. Experiment 1, however, recorded imprecision and bias where surplus identification required individuals to attend to just two sources (attribute magnitude and price); Experiment 2 added a third. The attributes of the hyperproducts were also designed to minimise perceptual error. Thus, even when attributes are small in number and perceptual representations are highly granular, surplus identification is limited. The mapping of attribute magnitudes and prices onto commensurate scales and the functional complexity of the mapping appear to be post-perceptual bottlenecks that more complete models of choice need to account for. This is not to say that limited attention and perceptual error are unimportant; products may contain attributes that are shrouded or hard to discern. Rather, there are capacity constraints in consumer choice even where relevant information is neither hidden nor fuzzy.

Secondly, our results imply important variation in consumers' capabilities across markets. Certain attribute-price relationships may warrant the description "complex" product, including those with non-monotonic attribute-price relationships. Models of *bounded rationality* often impose a high-level constraint on information processing that does not consider such variation. This approach is appealing because it can generate "predictions that do not depend on the details of how information is processed" (Sims 2003, p.666), which is beneficial for aggregate models of macroeconomic outcomes. Yet the scale and variability of inaccuracy that we report has microeconomic significance. Where the random error in random utility models is large, welfare estimates are rendered imprecise (Petrin, 2002). Microeconomic models therefore need to incorporate the drivers of variability in imprecision in consumers' assessments of surplus.

Thirdly, choice between options that do not involve explicitly probabilistic or uncertain future outcomes is generally considered "riskless" choice. In contrast, high imprecision in surplus identification for simple products indicates that it may be invalid for choice models to assume that the purchase of apparently simple products is risk-free. Gul et al. (2014) demonstrate that when limited cognition is built into a competitive equilibrium framework, consumption becomes a riskier prospect. Assuming that consumers have some awareness of the limitations we document, risk-aversion may affect market activity. Willingness to search and switch may be diminished not only by perceptions of the potential surplus in the market but also by how consumers view their ability to locate it. Some empirical comparisons of the potential for gains and the extent of consumer search indicate implausibly high search costs (Woodward and Hall, 2012). Consumers' concerns about their ability to harvest the gains may help to explain these disparities, requiring adaptation of relevant choice and search models.

Fourthly, recent models of industrial organisation that incorporate bounded rationality focus on the potential for firms to profit by actively confusing consumers, with the potential for welfare losses to be sustained in equilibrium (see Section 2.2). Much of this work concentrates on markets where firms obfuscate prices, generally by splitting full prices into more complex components. Of course, full prices usually can be determined objectively, while surpluses usually cannot. Nevertheless, the present experiments imply scope for firms to manipulate the complexity of product attributes as well as prices, with similar potential impacts on consumer search and equilibrium outcomes.

Finally, the substantial over- and under-estimation of surplus across the price range merits further consideration. Again, the finding echoes some studies of perceptual decision-making, in which simultaneous underweighting and overweighting of stimuli across a range survive extensive practice and feedback (De Gardelle and Summerfield, 2011; Michael et al., 2015). Although such decision-making may initially appear to be incompatible with an economic optimisation framework, Summerfield and Tsetsos (2015) argue that precision and bias may trade off within a neural system that seeks to optimise decisions given limited capacity. Barlow's (1961) "efficient coding hypothesis" posits that the limited range of neural signals requires a process of normalisation to specific decision-making contexts. Normalisation increases the discriminability of neural signals near the middle of a range, but at the cost of compressing, and thus biasing, signals towards the ends. Further research is needed to investigate whether the bias we report reflects such efficient coding and, if so, what contextual factors determine the normalisation process. Yet this explanation is consistent with our finding of reduced precision when the trade-off between two attributes is larger. The key point is that a strong and persistent bias may, in principle, result from an optimal trade-off given limited processing capacity.

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	520	Predicting International Higher Education Students' Satisfaction with their Study in Ireland Mairead Finn and *Merike Darmody*
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