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## **A Computational Theory of Exchange:**

Willingness to pay, willingness to accept  
and the endowment effect

*See revised version ([Working Paper No. 477](#))*

## **A Computational Theory of Willingness to Exchange**

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*Abstract:* We present a new theory of exchange and offer an alternative explanation for the endowment effect. Unlike neoclassical and reference-dependent theories, we do not base our approach on preference structure, but on optimal trading rules when value is perceived with error. We combine assumptions from economics and perceptual theory to develop a highly generalised formal model, in which agents treat their own perceptual error as a signal regarding the variability of future bids and offers. Optimising agents, with no aversion to risk or loss, produce an endowment effect that increases when goods are harder to value, in line with evidence.

*Keywords:* Endowment effect, willingness to accept, willingness to pay, exchange, uncertainty

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# **A Computational Theory of Exchange:**

## **Willingness to pay, willingness to accept and the endowment effect**

### **1. Introduction**

Exchange of ownership is a fundamental economic process. Yet numerous studies reveal that people approach simple exchanges in a manner that is not easy to square with microeconomic theory. Devising an alternative or additional theory that can adequately account for real exchange behaviour is, therefore, of fundamental importance. We present a theory that departs from prevailing explanations. Rather than viewing exchange behaviour as determined by the structure of preferences, we derive a formal model based on perceptual limitations inherent in the process of exchange.

For several decades it has been noted that contingent valuation studies involving environmental and public goods often result in large disparities between the prices people give when asked to state the maximum they are willing to pay (WTP) for a good and those they give when asked to state the minimum they are willing to accept (WTA) to give up the same good. Beginning with Kahneman, Knetsch and Thaler (1990) and Knetsch (1989), similar results have been recorded in laboratory experiments in which subjects exchange ordinary consumption goods such as mugs, pens and chocolate bars. Typically, WTA exceeds WTP by a factor of two or three, and subjects are unwilling to trade goods they own for goods they would prefer if offered a binary choice. Following Thaler (1980), this finding is known as the “endowment effect”.

For economic theory, these findings are no small matter. Most straightforwardly, the lack of an agreed explanation implies that we may not adequately understand the process of everyday exchange. The effect also questions the basis of neoclassical staples such as Hicksian consumer theory and the Coase Theorem, while raising the possibility that agents in markets routinely miss out on beneficial trade.

An alternative theoretical approach is provided by models that treat individual preferences over goods, characteristics of goods, or even utility itself, as “reference-dependent”, meaning that preferences change according to current endowments or expectations (Kahneman and Tversky, 1979; Tversky and Kahneman, 1991; Sugden, 2003; Köszegi and Rabin, 2006). Common to these models is the notion that people possess some form of fundamental aversion to loss, defined relative to a reference point.

While reference-dependent models provide a possible explanation for large disparities between WTA and WTP, they are challenged by other empirical work that questions the robustness of the endowment effect. Franciosi et al. (1996) adapted the original Kahneman et al. (1990) experiment by changing the experimental instructions, such that references to “buying”, “selling” and “price” were replaced by references to “choosing”. This simple manipulation reduced the endowment effect. When Shogren et al. (1994) elicited valuations of goods via repeated second-price auctions, the WTA-WTP disparity rapidly disappeared. Plott and Zeiler (2005, 2007) made other changes to experimental procedures that reduced or removed the endowment effect. In the 2005 study, subjects were given “extensive instruction” on the BDM value elicitation mechanism (Becker, DeGroot and Marschak, 1964) prior to the experiment, during which specific examples were “used to illustrate why announcing valuations that are not actual valuations is a dominated strategy” (Plott and Zeiler, 2005, p.537). In the 2007 study, it is not obvious which of many manipulations were most influential, but for a range of different conditions the endowment effect for the direct exchange of two goods varied in strength. Lastly, in a series of field experiments, List (2003, 2004) found that the endowment effect was stronger for (and perhaps confined to) less experienced traders at collectors’ trade fairs. These findings do not sit easily with the claim that the endowment effect is due to changes in preferences, ultimately caused by a fundamental aversion to loss, unless a number of auxiliary assumptions determine when and to what degree such changes in preferences take place.

Although a satisfactory explanation for these exchange experiments remains elusive, it would nevertheless be premature to conclude that the endowment effect is no more than an experimental artefact, or that similar phenomena do not occur in real markets. Numerous studies have reported large disparities between WTA and WTP in a wide

range of circumstances. Furthermore, the extent of the disparity varies systematically with the type of good. Horowitz and McConnell (2002) review 45 studies of WTA-WTP gaps and the endowment effect. They find that despite variation in methodologies across studies the ratio of WTA to WTP is systematically related to the type of good. Mean ratios are ten for public goods, non-market goods and resources related to health and safety; between two and three for ordinary consumption goods; just over two for lotteries; and just less than two for time. The authors conclude that the further away the good is from quantifiable money and the less routinely it is exchanged, the greater the WTA-WTP disparity is likely to be. Thus, while the endowment effect appears to be sensitive to aspects of the trading environment and the experience of the trader, it nevertheless represents a systematic and replicable behaviour. People's inclination is to set WTA much higher than WTP; more so the less easy it is to discern the value of the good.

The present paper offers a new account of exchange behaviour based on an alternative theoretical approach. We present a highly generalised formal model of exchange, based on assumptions that aim to capture important features of exchange activity in real markets. Our model determines WTA and WTP, such that either the presence or the absence of a disparity between the two amounts to a special case that can be linked to specific properties of the agent or the environment.

The most important assumption we make is that there is significant variability in the distributions of bids and offers for goods in markets. This assumption has strong empirical backing from several decades of research showing that price dispersion in product markets is ubiquitous (see Baye, Morgan and Scholten, 2006, for review). Given this, an agent who aims to optimise their surplus in exchange must consider the perceived value of the good and the perceived distribution of future bids and offers. With respect to this process, we make two psychophysical assumptions: the extent of error in the perceived value of goods is correlated across agents, and agents are able to use their own degree of error as a signal regarding the likely distribution of the valuations of other market participants. We support these two assumptions with psychophysical evidence. Agents then set WTA and WTP with a view to sequentially accepting or rejecting future bids and offers respectively, where the time and/or effort required to obtain bids and offers accumulates cost. Given these assumptions, which

are designed to approximate the exchange environment in the real economy, agents have the simple goal of maximising surplus from transactions. We require no assumptions about preferences over risk or loss.

Hence, our model links standard economic assumptions about the desire to optimise gains from trade with sound psychophysical assumptions about human perception of value. We then provide a formal derivation of WTA and WTP. The result is an endowment effect, which increases with the degree of perceptual error. This primary result requires agents to solve a fairly complex optimisation problem, but we also show how feedback over repeated trades may have heuristic value in learning to set optimum WTA and WTP.

The intuition behind our model is that when agents are themselves unsure of the value of a good, it signals that there is likely to be greater dispersion of valuations across potential trading partners. Put simply, if you find a good hard to value, then it is likely that others will too. Thus, there is an increased probability of obtaining a higher selling price and, similarly, an increased probability of obtaining a lower buying price, leading an optimising trader to raise WTA and lower WTP. Such consideration of the extent of price dispersion is part of the process of buying and selling in real markets. Therefore, if subjects in laboratory experiments and contingent valuation studies behave as if they are in a real market, they will produce a WTA-WTP disparity, provided the perceived value of the good is subject to error. Price dispersion is not generally present in experiments and, more importantly, the degree to which this is made clear to participants varies. It is this, we suspect, that explains why certain manipulations cause the endowment effect to disappear.

Section 2 provides necessary support and motivation for our assumptions, drawing on both economics and psychophysics. Section 3 presents the formal model. Section 4 relates the model to empirical findings on exchange. Section 5 concludes.

## 2. The perception of value

### 2.1 Price dispersion

Real product markets involve considerable price dispersion. Baye et al. (2006) review more than 30 empirical studies spanning a century of data and covering a large variety of consumer products. While there is great variation in the extent of dispersion, it is generally substantial and price differences in excess of 30% are common, even in competitive markets for homogeneous consumer goods. Furthermore, there is no indication that price dispersion has diminished in modern times, despite near costless price comparison.

These findings are important for exchange behaviour. Substantial price dispersion suggests that perceptions of possible exchanges are imprecise. Furthermore, if price dispersion characterises real markets, then agents should be likely to have adapted to it. To maximise surplus from transactions, they may take account of price dispersion signals when determining prices at which they are willing to trade.

### 2.2 Perceptual discrimination and exchange

When deciding whether to trade one object for another, an agent needs to discriminate which has the higher value (to them). In other words, the agent must perceive the value of what is obtained and compare it to a perception of the value of what is given up. An exchange, therefore, parallels a forced-choice discrimination task. Such tasks are routinely used to measure the precision of human perception and much is understood about how the humans process them. Perceptual theory and empirical findings may therefore be relevant to the process of exchange.

Briefly, in a standard perceptual discrimination task, subjects make repeated forced-choice comparisons between different test stimuli and a reference stimulus. The most common estimate of discrimination is the Weber fraction,  $\Delta S/S$ , where  $S$  is the length, loudness, weight etc. of the reference stimulus and  $\Delta S$  is the “threshold” or “just noticeable difference” that can be detected reliably (i.e. the subject can perceive that  $S - \Delta S < S < S + \Delta S$ ). Forced-choice experiments estimate the probability of correctly

judging whether the test stimulus is longer, louder, heavier etc. than the reference stimulus for a range of test stimuli. The threshold,  $\Delta S$ , is frequently defined as the standard deviation of the cumulative normal distribution that best fits the data. For many perceptual tasks, a cumulative normal provides a good fit and the resulting Weber fraction is approximately constant over a wide range of  $S$  (the Weber-Fechner Law). This experimental method can be applied not only to perceptual primaries, such as length, loudness or heaviness, but to much higher-level perceptions, including complex patterns and time. It is also successful where subjects must compare an immediate stimulus to a perception held in memory. There is no reason why, in principle, it cannot be used to investigate perceived value.

The analogy between exchange behaviour and forced-choice perceptual discrimination is instructive. Valuation is a complex perceptual task, likely to involve immediate perceptions of the good and perceptions of past experiences of the good or similar goods held in memory, which may be influenced by many other sources of information. Yet, ultimately, there must be some internal representation of value that humans use to compare the value of a good against the value of other goods, or against the value of monetary amounts. We can therefore ask to what extent this internal representation is subject to error and whether the extent of error can be taken into account when making judgements.

A number of previous studies have raised the issue of uncertainty of value in relation to the endowment effect. Loomes, Orr and Sugden (2009) in particular incorporate “taste uncertainty” into their model, such that agents take into account multiple future “taste states”. Our concept of perceptual error is similar, in that it is a form of uncertainty generated internally by the way humans process information, whereas uncertainty in economics is more usually thought of as possible future states of the economic environment. Yet our concept of perceptual error also differs, because we see the uncertainty not as the product of different tastes, experiences or moods, but as reflecting a fundamental limitation on the accuracy of human perception. This distinction is crucial to our model, because it implies that the extent of perceptual error in valuing different goods is likely to be correlated across individuals. While different people may have different expertise at assessing the value of goods to be

exchanged, there is also likely to be much commonality: some goods are simply harder to value than others, whatever one's tastes.

### *2.3 Psychophysical evidence*

The extent of error surrounding many perceptual dimensions is empirically established. In vision, Weber fractions for the discrimination of basic spatial dimensions such as size or length are around 3 – 8% (e.g. Burbeck, 1987), i.e. a stimulus must be 3 – 8% longer for it to be reliably perceived as such. Weber fractions for perceiving contrast are higher, at 10 – 20% (Legge, 1981), while for more complex properties of properties of three-dimensional shapes they may be as high as 15 – 30% (Lunn and Morgan, 1997). For discriminating the weight of an object held in the hand, Weber fractions are typically around 10% (e.g. Brodie and Ross, 1984). These levels of performance are obtained by trained observers after undertaking practice specific to the perceptual task, which improves on initial performance (Fiorentini and Berardi, 1980). Yet individual differences in performance are generally small relative to the variation in performance across tasks. In summary, therefore, even the perception of basic visual and haptic dimensions is subject to significant degrees of error that vary systematically according to the task.

The apparently straightforward process of valuing a coffee mug requires an individual to judge not only perceptual basics like its size and weight, but also complex perceptual properties like attractiveness and durability, before even taking account of perceptions of socially influenced factors that affect value, such as fashionability. How accurately people can discriminate on such dimensions is not known. However, given the magnitude of error surrounding basic perceptual dimensions, our internal representations of value are likely to be subject to significant perceptual error, reflecting fundamental limitations of human perception that are common to us all.

Turning to the second question, there is evidence that people can estimate the extent of their own perceptual error and incorporate that estimate into judgements. For instance, it is possible to compare performance in assessing the shape of an object by vision, by touch and simultaneously by both vision and touch, where the observer must combine information from both senses to reach a judgement. In performing such



tasks, people weight the information from the two different perceptual systems inversely according to the degree of error associated with each perceptual system, as measured by performance in the tasks using vision or touch alone. Subjects' judgements resemble the outcome of maximum likelihood estimation (Ernst and Banks, 2002).

Thus, evidence supports our two psychophysical assumptions. First, perceptual error when valuing goods is likely to be large and to vary systematically with the type of good, such that the degree of error will be correlated across individuals. Second, people are likely to be able to take account of the degree of error in judgements.

#### *2.4 Adaptive setting of WTA and WTP*

Using the above evidence as a basis for model assumptions, we aim to show how the endowment effect may result from an adaptive response to the challenge of conducting successful exchanges, rather than from an irrational psychological quirk, misconception or arbitrary aversion. In doing so, the blending of economic theory and perceptual theory extends beyond drawing on concepts and evidence from both fields. There are also commonalities of theoretical approach.

While the present paper develops a theory based on optimisation, in the orthodox economic tradition, it also seeks a “computational theory” of exchange, as envisaged by David Marr’s groundbreaking contribution to neuroscience and psychology. Marr argued that perception needed to be understood through “computational theories” or “functional descriptions of what information processing systems, including brains, are designed to do” (Marr, 1982). Computational theories explain not only what the system does, but why it makes sense for it so to do. This distinction has parallels with the distinctions between normative and positive economics and between adaptive and non-adaptive traits in evolutionary science. We call our contribution a computational theory, however, because we do not wish the model to be regarded as normative and because we do not explicitly model an evolutionary process. Nevertheless, the theory does more than describe observed behaviour; it proposes a rationale for it.

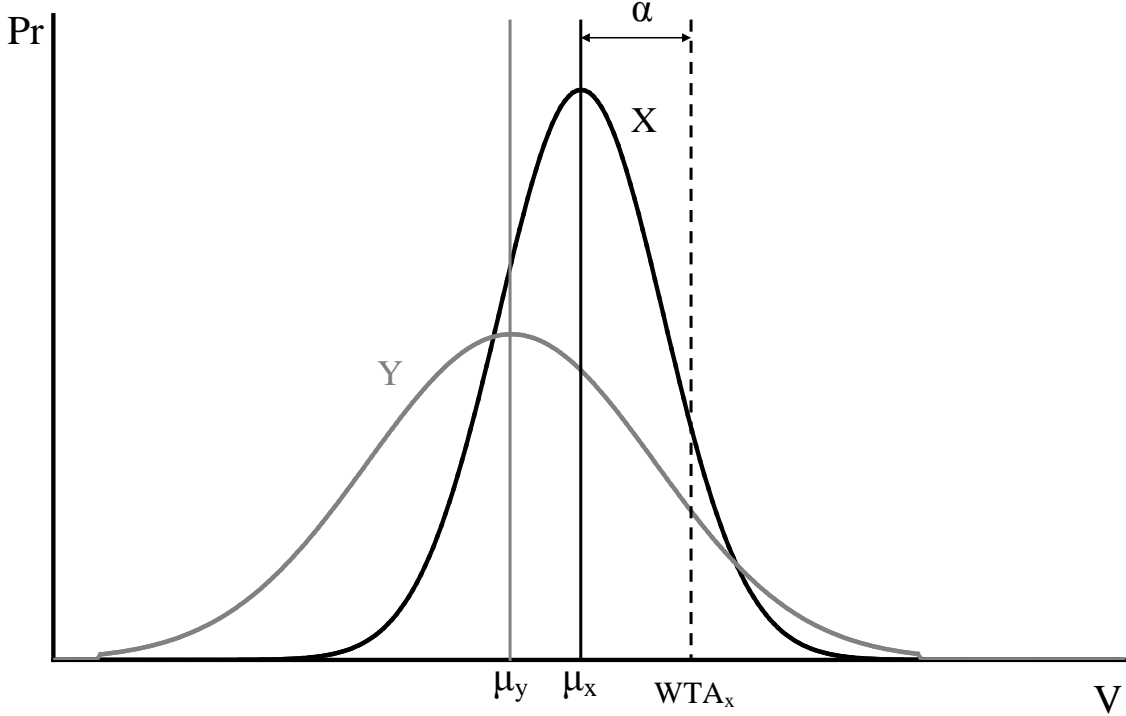
Exchange in real markets is a process of interaction with other agents involving the possibility of a series of encounters in which bids and offers may be accepted or rejected. Thus, an agent's WTA or WTP determines not only their potential surplus, but also the likelihood that they can trade. Provided there is price dispersion, which there usually is, an adaptive setting of WTA and WTP will resolve the trade-off between holding out for a higher surplus and reducing the likelihood of encountering a willing trading partner. In such a trade-off, any signal regarding the distribution of future bids or offers, including perceptual error in valuation, is useful information.

### 3. Model

The basic set-up for the model is depicted in Figure 1, where we consider an agent who must decide their WTA. The derivation of the optimal WTA applies to the analogous case of the optimal WTP by similar argument. We assume that an agent's perceptual representation of the value of each good consists of a continuous probability density function over a range of possible values. Perceptual error is considerable, such that variabilities are relatively large with respect to expected values. The agent is endowed with a good, which they perceive to be of value  $X \sim N(\mu_x, \sigma_x^2)$ . The agent must also perceive the distribution of bids they can obtain for the good. We make the simplifying assumption throughout that bids correspond to quantities of money, the values of which are represented without error. Moreover, we assume that the mapping of perceived value to numerical amounts is perfect, such that WTA itself is represented without error. Neither simplifying assumption will hold in reality, but we anticipate that the variabilities involved are small relative to the perceptual error that underpins the model. Thus, we can directly compare the perceived value of the good,  $X$ , with the perceived value of future bids,  $Y \sim N(\mu_y, \sigma_y^2)$ , on the same dimension of value.

Note that while the assumptions of normality represent a special case which is useful for illustration, the main result we derive holds for all continuous distributions (see below and Appendix). Furthermore, in Figure 1,  $\mu_y > \mu_x$  and  $\sigma_y > \sigma_x$ , but these are not necessary for our main result.

**Figure 1: Model set-up for willingness to accept (WTA).** The agent owns good of perceived value  $X$ , perceives sequential future bids of value  $Y$ , each costing  $c$ , and aims to set WTA to maximise surplus.



Based on these perceptions, the agent sets their minimum acceptable price for selling the good,  $WTA = \mu_x + \alpha$ . We assume that the agent aims to maximise expected surplus.

Given commonalities of perceptual limitations, the extent of perceptual error surrounding the agent's valuation,  $\sigma_x^2$ , will be positively correlated with the perceptual error of other market participants. Our main conjecture is that the agent will take this correlation into account in their representation of the distribution of bids they are likely to receive. The nature of the correlation might be considered a measure of the agent's level of expertise in dealing with the good, but we do not include variation in expertise in the model. The variability in the agent's perception of likely bids will also be determined by other factors, such as their perception of variation in tastes or needs. Thus, our formulation is  $\sigma_y^2 = f(\sigma_x^2, \tau_x^2)$  where  $\tau_x^2$  captures the

perceived variability that is not due to perceptual error, with  $f_{\sigma_x^2} > 0$  and  $f_{\tau_x^2} > 0$ .<sup>1</sup>

The relationship between  $\mu_x$  and  $\mu_y$  will depend on whether the perceived value of the good to the agent is more or less than the value they perceive it to have for others, and on the degree of surplus they expect bidders to build in to bids. It is not necessary to constrain either to obtain our results; nor is it desirable, since we are seeking a highly generalised result.

We assume that the agent expects to receive bids in sequence  $\{Y_1, Y_2, \dots, Y_i, \dots\}$  and sets WTA in advance of receiving bids.<sup>2</sup> Our model can be adapted easily to one in which the agent posts a selling price, but because empirical studies of the endowment effect usually elicit values for WTA and/or WTP, we set WTA in order to determine rejection or acceptance of a subsequent sequence of bids.

A vital assumption in our model is that receiving a bid is not costless. We assign a (small) cost,  $c$ , to receiving each bid, which we call the “encounter cost”. One way to conceive of the encounter cost is that the number of encounters in which bids are received determines the length of (costly) time it takes to make the sale, although other conceptions are possible, including equating it to a search cost. We assume, for the present model, that the encounter cost is perceived accurately.

Given these assumptions, we can formulate the expected surplus from the transaction for any given WTA. Assuming a sale is made, the agent expects to receive a price equal to the expected bid given that the bid is greater than WTA,

$$E(\text{price}) = E(Y | Y > \mu_x + \alpha) \quad (1).$$

In addition to giving up the good, the agent incurs encounter costs as a result of rejecting bids. In setting WTA, they determine a fixed probability of accepting a bid. Thus, the expected number of encounters required to make a sale conforms to a

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<sup>1</sup> A more precise definition of  $\tau_x^2$  is not essential for our results, although we suspect that  $\sigma_x^2$  and  $\tau_x^2$  will not be independent in reality. In particular, it seems likely that perceptual error surrounding valuation will be positively correlated with variation in tastes for the good.

<sup>2</sup> Clearly, it is possible for the agent to update their perception of the distribution of bids in light of the ongoing sequence of bids they receive, but for the present we do not incorporate updating in the model.

geometric distribution, with parameter  $\Pr(Y > \mu_x + \alpha)$ . The expected total encounter cost (up to and including making the sale) is therefore given by the encounter cost multiplied by the reciprocal of the probability of making a sale

$$E(\text{total encounter cost}) = \frac{c}{\Pr(Y > \mu_x + \alpha)} \quad (2).$$

Thus, the agent can be considered to face an optimisation problem, in which the aim is to maximise the expected surplus,  $E(S)$ , from selling the good. Combining equations (1) and (2), the agent chooses  $\alpha$  to maximise

$$E(S) = E(Y | Y > \mu_x + \alpha) - \mu_x - \frac{c}{\Pr(Y > \mu_x + \alpha)} \quad (3).$$

This optimisation problem represents a trade-off. Increasing  $\alpha$  increases the expected price, but also increases expected encounter costs.

Looking at (3), the structure of our model shares features with some models of consumer search, perhaps most notably that of Reinganum (1979). The model centres around a trade-off between expected price and incremental costs, where the total cost conforms to a geometric distribution. The similarity is instructive, but there are major differences too. Our model of exchange is much more general. It applies to selling as well as buying. The encounter cost need not be a search cost. Moreover, while consumer search models are concerned with deriving an equilibrium between the consumer's optimum search strategy and the firm's optimum pricing strategy, we are concerned with deriving adaptive buying and selling strategies for exchange, involving generalised distributions of perceived value, bids and offers, whether there are firms involved or not. With respect to the market, all that is necessary for our results is that there is some price dispersion linked to the extent of perceptual error in valuation. Contrary to most search models, we do not assume homogeneous buyers (or sellers) with perfect information regarding the distribution of offers (bids). Indeed, we consider this assumption unrealistic and instead conjecture that forming a perception of the distribution of offers (bids) is crucial to determining WTA (WTP).

The solution to the optimisation problem is derived for any continuous distribution in the Appendix. The existence of a positive  $\alpha^*$  that satisfies (3) depends on

$$c < \int_{\mu_x}^{\infty} (1 - F(y)) dy \quad (4)$$

where  $F(y)$  is the cumulative distribution function of  $Y$ . The condition specified by (4) makes sense: if the encounter cost is too high then there is no price at which a surplus is likely to be made. WTA is determined by  $\alpha^*$  which satisfies

$$c = \int_{\mu_x + \alpha^*}^{\infty} (1 - F(y)) dy \quad (5).$$

**PROPOSITION 1:** *For good  $X$  of expected value  $\mu_x$  subject to sequential bids,  $Y$ , each received at a small encounter cost  $c$ , with continuous cumulative distribution function  $F(y)$ , there exists  $\alpha^*$  such that willingness to accept,  $\mu_x + \alpha^*$ , maximises the expected surplus from exchange.*

Given (5),  $\alpha^*$  is decreasing in  $c$ . The higher the encounter cost the greater the need to obtain a sale from fewer encounters. The relationship between  $\alpha^*$  and  $\sigma_y$  is less straightforward, because it depends on the shape of the distribution of  $Y$ . In the Appendix we derive the following:

**PROPOSITION 2:** *For bids,  $Y'$ , with consistently higher probability  $Pr(Y' > y) > Pr(Y > y)$  for all  $y$  greater than a fixed value,  $k$ , an agent who maximises surplus will set a higher willingness to accept, such that  $\alpha^{*'} > \alpha^*$ .*

In other words, the fatter the upper tail of the perceived distribution of bids, the greater  $\alpha^*$  and hence the higher WTA. In all practically applicable cases, where the perceived distribution of bids is unimodal and continuous, with a steadily decreasing probability of receiving bids ever higher than the mean,  $\alpha^*$  is increasing in  $\sigma_y$ .

We can further derive the maximum expected surplus from setting WTA according to (5) so as to maximise (3), which turns out to have an interesting solution

$$\max E(S) = \alpha^* = WTA - \mu_x \quad (6).$$

More tellingly, in order for (6) to hold, it must also be the case that

$$E(\text{total encounter cost}) = E(\text{price}) - (\mu_x + \alpha^*) \quad (7).$$

**PROPOSITION 3:** *The expected total encounter costs of an agent who sets willingness to accept (WTA) to maximise surplus are equal to the expected price over and above WTA.*

From a mental accounting perspective, this solution regarding the expected surplus given an optimal setting of WTA is interesting, for at least two reasons. First and most straightforwardly, it gives a ready indication of the expected surplus, which may be of benefit to an agent involved in repeated trading activity. Second, the relationship specified in (7) between the expected price and the expected cost of bids may have heuristic value in helping to set WTA through experience. Equation (3) represents a complex optimisation problem, but in repeated buying and selling agents will get feedback that is suggestive of setting WTA too high or too low. If a seller repeatedly incurs higher encounter costs than the additional price they ultimately obtain, over and above WTA, then WTA is being set too high, and vice-versa.

This result is intuitively appealing. When selling, there are times when it takes so long (or so much effort) to obtain the sale that agents regret holding out for the higher price. Yet there are also times when a quick sale close to the minimum acceptable price leads agents to wonder whether they shouldn't have held out for more. On average, given (7), if agents balance the time and effort against the additional price obtained above WTA, they are optimising surplus.

Having solved (3) for the general case of a continuous distribution, we now consider the application to the normal distribution, which offers greater insight into the

properties of the solution. For  $Y \sim N(\mu_y, \sigma_y^2)$ , the solution to the optimisation problem (see Appendix) is such that

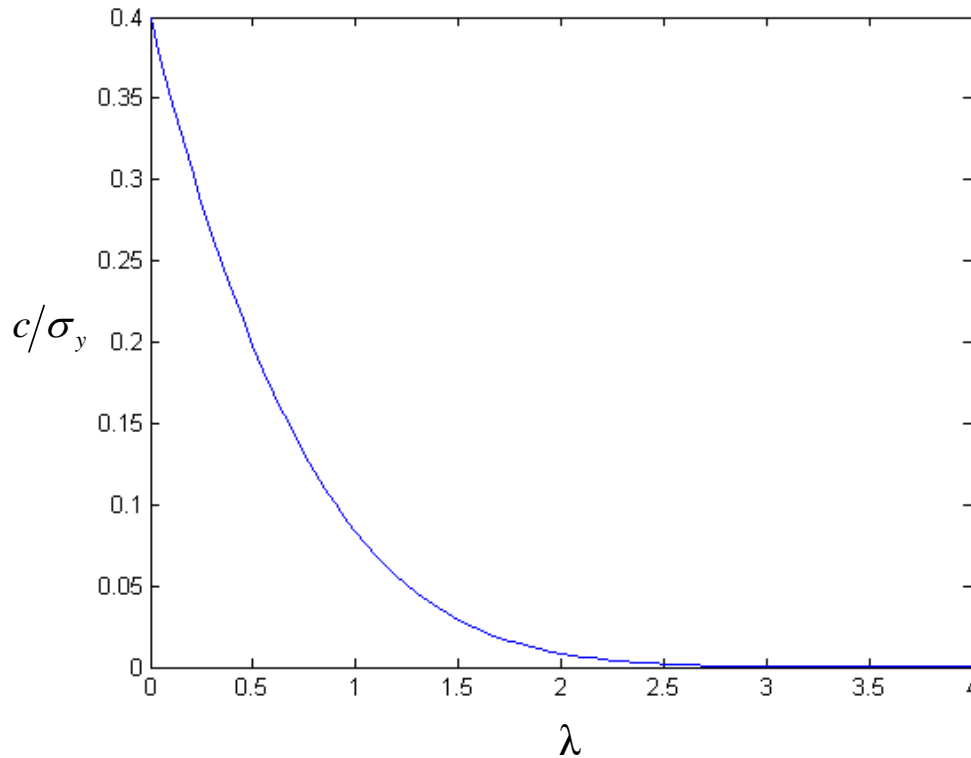
$$\frac{c}{\sigma_y} = \frac{1}{\sqrt{2\pi}} e^{-\frac{\lambda^2}{2}} - \lambda(1 - \Phi(\lambda)) \quad (8)$$

where  $\Phi$  is the cumulative distribution function of the standard normal distribution and

$$\lambda = \frac{\mu_x - \mu_y + \alpha^*}{\sigma_y} \quad (9).$$

Figure 2 shows the relationship between  $\lambda$  and  $c/\sigma_y$ , which is not intuitively obvious from (8) and (9).  $\lambda$  is increasing in  $\sigma_y$  and decreasing in  $c$ . Since  $\sigma_y^2 = f(\sigma_x^2, \tau_x^2)$  and  $f_{\sigma_x^2} > 0$ , the greater the degree of uncertainty in the perception of value, the greater WTA, while the higher the cost of encounters, the lower WTA.

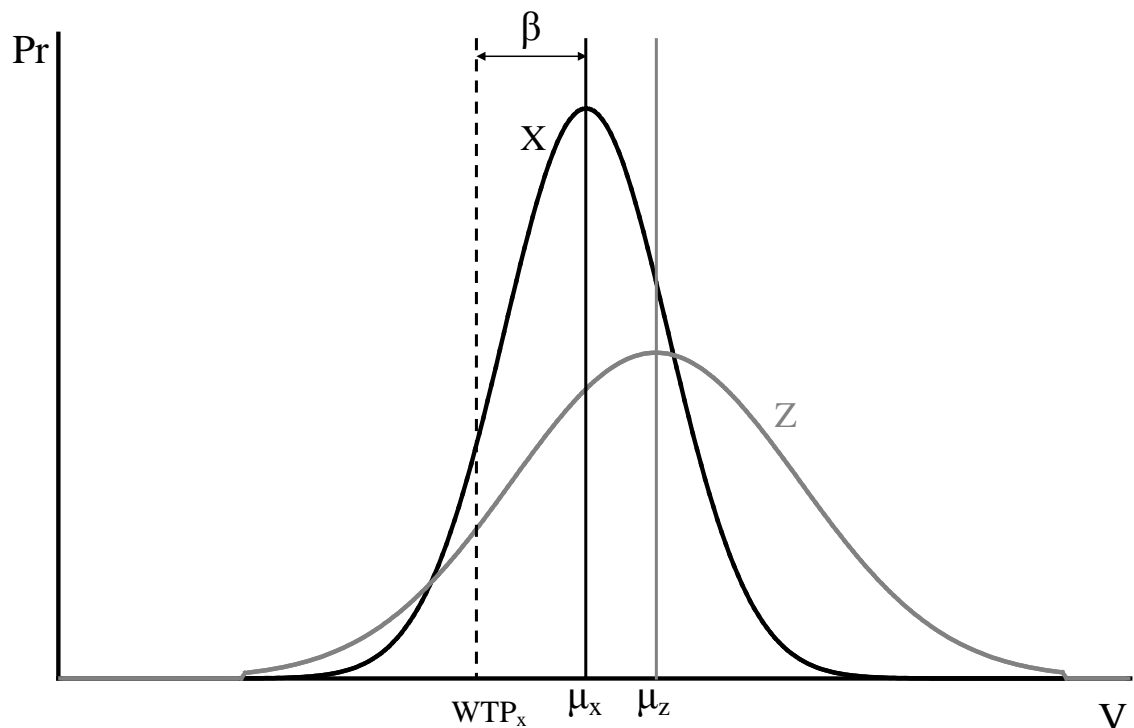
**Figure 2: Optimal willingness to accept (WTA) when perceptual error in valuation is normally distributed. WTA (which is increasing in  $\lambda$ ), is a decreasing function of the cost of sequential encounters in the marketplace,  $c$ , and an increasing function of the perceived variability of future bids,  $\sigma_y$ .**





Lastly, we consider the case of WTP, which is depicted in Figure 3. This time, the agent must set WTP for a good of perceived value  $X \sim N(\mu_x, \sigma_x^2)$ . We assume a perceived distribution of offers  $Z \sim N(\mu_z, \sigma_z^2)$  and that the agent receives a sequence of offers  $\{Z_1, Z_2, \dots, Z_i, \dots\}$ , each at an encounter cost,  $c$ .

**Figure 3: Model set-up for willingness to pay (WTP). The agent considers the purchase of a good of perceived value  $X$ , perceives sequential future offers of value  $Z$ , each costing  $c$ , and aims to set WTP to maximise surplus.**



The agent's optimisation problem is to choose  $\beta$  to maximise

$$E(S) = \mu_x - E(Z | Z < \mu_x - \beta) - \frac{c}{\Pr(Z < \mu_x - \beta)} \quad (10).$$

Similarly to the solution in the case of WTA,  $\beta^*$  satisfies (see Appendix)

$$c = \int_{-\infty}^{\mu_x - \beta^*} F(z) dz \quad (11)$$

and the expected surplus, given  $\beta^*$ , is given by

$$E(S) = \beta^* = \mu_x - WTP \quad (12).$$

Thus, the general solution is symmetrical to that for WTA and the equivalent of propositions 1 – 3 hold for WTP also. Thus, the greater the degree of perceptual error in valuation, the higher  $\beta$  and the lower WTP, while the higher the cost of encounters, the lower  $\beta$  and the higher WTP.

For the normal distribution we obtain:

$$\frac{c}{\sigma_z} = \eta \Phi(\eta) + \frac{e^{-\frac{\eta^2}{2}}}{\sqrt{2\pi}} \quad (13)$$

where

$$\eta = \frac{\mu_x - \mu_z - \beta^*}{\sigma_z} \quad (14).$$

This solution is again symmetrical to that for WTA, such that  $\eta$  is decreasing in  $\sigma_z$  and increasing in  $c$ .

Before relating the model to other findings and theories, it is important to note how few constraints it involves. Given our starting assumptions, the result that optimal WTA (WTP) is increasing (decreasing) with uncertainty in perceived value requires only that the encounter cost is small relative to the perceptual error. If so, an endowment effect will be a characteristic of optimising traders.

#### **4. Relationship to empirical findings**

Most obviously, our model provides an explanation for the existence of an endowment effect, the strength of which will vary with the type of good (c.f. Horowitz and McConnell, 2002). The explanation requires that value is perceived less accurately as the good concerned moves from money to time, to lotteries, to consumer goods and to non-market or public goods. This relationship between the WTA-WTP gap and the type of good might plausibly be subject to a more stringent empirical test,

whereby the strength of the endowment effect is compared to an independently derived measure of the accuracy of valuation for different goods within the same study.

Our theory implies a specific understanding of the cause of the standard laboratory result (Knetsch, 1989; Kahneman et al., 1990). If subjects respond to the experimental environment as if they are engaging in typical trade outside the laboratory, they will instinctively set WTA higher than WTP, the more so the harder they find the good to value. This behaviour appears to be irrational only because the experimental market institution is a one-shot game in which there is no price dispersion. Real markets are not one-shot games in which the final outcome depends on whether a single announced price falls above or below a threshold, or where immediate failure to trade implies permanently lost opportunity. A behaviour that is well adapted to real markets may appear nonsensical if it is not adjusted to match an artificial one-shot/one-price market, which terminates instantly at an exact price.

Some of the various experimental manipulations that lead to the reduction or removal of WTA-WTP disparities support this account. Manipulations that emphasise the one-shot nature of the experiment, or that induce subjects to treat the problem as one of choice rather than trade, will be inclined to remove the endowment effect for at least some of the subjects. Perhaps the most straightforward illustration of this is the reduction in the WTA-WTP gap that occurred when Franciosi et al. (1996) replaced references to “buying”, “selling” and price with references to “choosing” in the experimental instructions. We suspect that a similar process explains the findings of Plott and Zeiler (2005, 2007). Almost all of the many experimental manipulations involved in the studies, especially training in the logic of the BDM value elicitation mechanism, would have been likely to break the link between behaviour in the laboratory and the way WTA and WTP are usually determined in real markets. In simple terms, if experimenters point out at length and with examples that people’s instinctive setting of WTA and WTP will backfire in a one-shot experimental market without price dispersion, it is very likely to cause them to change behaviour. Similarly, in Plott and Zeiler (2007), the “full set of controls” condition replaced the usual practice of inviting subjects to trade the good they owned for another good, and employed instead a decision form asking them to circle the item they wished to take

home. A theory of exchange in real markets will only apply if experimental procedures lead subjects to behave as they would outside the laboratory, either because that is their instinctive response to the procedure, or because that is what they believe they are being asked to do. Thus, while we are inclined agree with Plott and Zeiler's conclusion that the endowment effect may not result from changes in preferences caused by ownership, we are not convinced that it results from what they term "subject misconceptions". Our interpretation is straightforward: if subjects believe they are being invited to make a once-off choice, they will choose what they think they prefer, but if they behave as if in a normal market they will be inclined to set WTA well above WTP.

The biggest point of difference between our computational theory of exchange and previous accounts of the endowment effect is that the theory depends on not only how agents value goods, but also on their perception of future bids and offers. This central claim is also consistent with evidence. Experimental manipulations that affect the distribution of bids and offers alter the WTA-WTP gap. If the value elicitation mechanism is an auction, where bids and offers are irrelevant to the likelihood of exchange, the endowment effect disappears over a few rounds (Shogren et al. 1994). When a uniform price double-auction is employed, which provides direct feedback about the distribution of bids and offers by posting the latest high bid and low offer, the endowment effect is reduced (Franciosi et al., 1996). Lastly, McConnell and Horowitz (2002) note the initially counterintuitive finding that WTA-WTP gaps tend to be larger in studies that use an incentive compatible value elicitation technique such as BDM . We suspect that by presenting subjects with a list of prices, usually uniformly distributed between generous upper and lower bounds, the mechanism generates a similarly generous signal about the experimenters' expectations regarding variability in valuations. If subjects respond to this signal then they will exhibit a larger WTA-WTP disparity, according to our model.

The results of List's (2003, 2004) field experiments, in which the endowment effect was absent for experienced dealers at a sports card market, are more problematic. Presumably, valuations of experienced dealers are likely to be subject to less perceptual error than those of inexperienced dealers. But in our model, perceptual error acts via its correlation with the degree of variability in bids and offers, and it is

not immediately obvious whether this correlation would be stronger or weaker for experienced dealers. On the other hand, experienced dealers are likely to have higher encounter costs, because List defined experience by the number of trades a dealer routinely made. Higher encounter costs reduce the endowment effect, according to our model. Still, List's most striking result, and the most difficult to explain, is that experienced sports card dealers did not display any endowment effect in a standard experiment involving mugs and candy bars (List, 2004). This result may reflect several factors associated with experience that our model links to the endowment effect: more accurate perceptions of value, higher encounter costs, and a greater likelihood of understanding the unusual one-shot/one-price market institution. While possible, this explanation is unsatisfactorily general.

## **5. Conclusion**

Much of the literature on WTA-WTP disparities and the endowment effect centres on whether neoclassical theory or reference dependent theories offer the best account of exchange behaviour. These theories differ regarding the shape of preference functions, but are similar in other respects. Both assume that willingness to trade is determined by whether a potential trade increases utility, given the shape of preferences. Our computational theory of exchange departs from this debate, because it focuses not on preferences over outcomes but on the process of exchange itself.

We combine economic and perceptual theory into a highly generalised model. We show that under relatively simple and realistic assumptions, where markets involve price dispersion and sequential encounters, and where value is perceived with significant error, optimal traders will set WTA well above WTP; more so the greater the difficulty involved in valuing the good. Thus, the endowment effect may reflect the fact that substantial perceptual error plays a decisive role in people's willingness to exchange, whatever their preferences. We also show how, in this uncertain environment, comparison of prices ultimately paid and the ease or difficulty of encountering trading partners has heuristic value for agents aiming to optimise WTA and WTP.

Our theory can account for the standard experimental findings and the association of the endowment effect with different types of good. Its validity as an explanation requires, however, that adaptive behaviour in real markets carries over into laboratory experiments where it is evidently suboptimal. Thus, rather than considering the endowment effect to be a laboratory finding that may not occur in the real economy, we contend that it is more likely to be a real world phenomenon that is sometimes absent in the laboratory. Consequently, while the disparity between WTA and WTP may be a characteristic of behaviour in real markets, the under-trading that occurs in artificial one-shot/one-price laboratory markets may not be a factor in ongoing markets with price dispersion.

Whether or not our computational theory proves to be a good account of exchange behaviour, there is a larger point to be made. Neoclassical theory and reference dependent theories largely ignore the dimension of skill involved in exchange activity. To do so is to do more than to assume that individual differences in trading ability are of secondary importance. Given the uncertainties and complexity involved, the ability of humans to exchange goods successfully and thus continually to reap the benefits of gains from trade requires explanation in its own right.

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## Appendix

### *Proof of Proposition 1*

Require  $\alpha$  to maximise

$$E(S) = E(Y | Y > \mu_x + \alpha) - \mu_x - \frac{c}{\Pr(Y > \mu_x + \alpha)}.$$

Suppose that  $Y$  has density function  $f(y)$  and cumulative distribution function  $F(y)$ . Then

$$\begin{aligned} E(Y | Y > \mu_x + \alpha) &= \left( \int_{\mu_x + \alpha}^{\infty} y f(y) dy \right) / (1 - F(\mu_x + \alpha)) \\ &= \left( \left[ -y(1 - F(y)) \right]_{\mu_x + \alpha}^{\infty} + \int_{\mu_x + \alpha}^{\infty} (1 - F(y)) dy \right) / (1 - F(\mu_x + \alpha)) \\ &= \mu_x + \alpha + \left( \int_{\mu_x + \alpha}^{\infty} (1 - F(y)) dy \right) / (1 - F(\mu_x + \alpha)). \end{aligned}$$

Hence

$$E(S) = \alpha + \left( \int_{\mu_x + \alpha}^{\infty} (1 - F(y)) dy - c \right) / (1 - F(\mu_x + \alpha)).$$

Differentiate to find  $\alpha^*$

$$1 - (1 - F(\mu_x + \alpha)) / (1 - F(\mu_x + \alpha)) + f(\mu_x + \alpha) \left( \int_{\mu_x + \alpha}^{\infty} (1 - F(y)) dy - c \right) / (1 - F(\mu_x + \alpha))^2 = 0.$$

So  $\alpha^*$  satisfies

$$c = \int_{\mu_x + \alpha^*}^{\infty} (1 - F(y)) dy.$$

Calculation of second order conditions indicates that the second derivative of  $E(S)$  with respect to  $\alpha$  is negative and thus that this is a maximum.

Considering the expression for  $E(S)$  we can see that

$$\max E(S) = \alpha^*$$

(and hence *Proposition 3*) provided that

$$c < \int_{\mu_x}^{\infty} (1 - F(y)) dy.$$

Note we are assuming that  $F(y)$  is continuous and strictly increasing on the range  $(a, b)$  of permitted values for  $Y$  and that  $y(1 - F(y)) \rightarrow 0$  as  $y \rightarrow \infty$ .

### *Proof of Proposition 2*

Suppose that  $Y', Y$  have cumulative distribution functions  $F_u, F$  which satisfy the following: for some  $k > 0$  and for all  $y > k$ ,  $F_u(y) < F(y)$ . This implies that  $Y'$  has a fatter upper-tailed distribution than  $Y$ , since  $\Pr(Y > y) = 1 - F(y)$ , similarly for  $Y'$ . Then

$$\int_{y_1}^{\infty} (1 - F_u(y)) dy - \int_{y_1}^{\infty} (1 - F(y)) dy > 0, \text{ whenever } y_1 > k.$$

Thus for solutions  $\alpha^*, \alpha^*$  such that

$$c = \int_{\mu_x + \alpha^*}^{\infty} (1 - F_u(y)) dy = \int_{\mu_x + \alpha^*}^{\infty} (1 - F(y)) dy,$$

it must hold that  $\alpha^* > \alpha^*$ , whenever  $\mu_x + \alpha^* > k$ .

Note that for normal distributions with the same mean,  $k$  can be taken to be the mean, and furthermore having a fatter upper-tailed distribution is equivalent to having a greater variance.

### **Application to normal distribution:**

#### *Willingness to Accept (WTA)*

Solve for  $\alpha^*$  if

$$\frac{c}{\sigma_y} < \int_{\frac{\mu_x - \mu_y}{\sigma_y}}^{\infty} (1 - \Phi(z)) dz$$

(using transformation to standard normal  $\Phi \sim N(0,1)$  knowing that  $Y \sim N(\mu_y, \sigma_y^2)$ ).

Then to find  $\alpha^*$  we use

$$\frac{c}{\sigma_y} = \frac{1}{\sqrt{2\pi}} e^{-\frac{\lambda^2}{2}} - \lambda(1 - \Phi(\lambda)) \quad , \text{ where } \lambda = \frac{\mu_x - \mu_y + \alpha^*}{\sigma_y} .$$

When this is solved the expected surplus is

$$E(S) = \alpha^* .$$

*Willingness to Pay (WTP)*

Suppose amount to pay is  $X \sim N(\mu_x, \sigma_x^2)$ , and that offers are distributed as  $Z \sim N(\mu_z, \sigma_z^2)$ . Suppose WTP is  $\mu_x - \beta^*$ , which satisfies a similar equation, that is  $\beta^*$  maximises expected surplus

$$E(S) = \mu_x - E(Z | Z < \mu_x - \beta) - \frac{c}{\Pr(Z < \mu_x - \beta)} .$$

With similar computations to WTA case we determine  $\beta^*$  according to

$$c = \int_{-\infty}^{\mu_x - \beta^*} F(z) dz ,$$

giving  $E(S) = \beta^*$ .

For the normal distribution we get

$$c = \sigma_z \int_{-\infty}^{\eta} \Phi(z) dz$$

$$\text{so that } \frac{c}{\sigma_z} = \eta\Phi(\eta) + \frac{e^{-\frac{1}{2}\eta^2}}{\sqrt{2\pi}} , \text{ where } \eta = \frac{\mu_x - \mu_z - \beta^*}{\sigma_z} .$$